Solution to Problem 5.3

Applying flow balance, we get

$$0.6 l_3 = l$$
$$0.5 l_1 = l_3$$
$$0.5 l_1 + l = l_2$$
$$0.8 l_2 + 0.2 l_3 = l_4$$

which yields l_1=3.3333l, l_2=2.6667l, l_3=1.6667l, l_4=2.4667l as the throughputs of the individual queues Q_1, Q_2,, Q_3 and Q_4.

Defining $r = l/m$, the corresponding values of the traffic offered to the four queues are r_1=3.3333r, r_2=5.3334r, r_3=1.6667r and r_4=4.9334r. Obviously, the system will be stable only if the largest value of the traffic offered to a queue is less than unity. This implies that the maximum value of l for which the queueing network will be stable will be *0.1875m* corresponding the traffic offered to Q_2.

For the specific values of l=*0.1* and m=*1*, we get r=*0.1* and the state distribution will be given by

$$P(n_1, n_2, n_3, n_4) = (0.66667)(0.46666)(0.83333)(0.50666)(0.33333)^{n_1} (0.53334)^{n_2} (0.16667)^{n_3} (0.49334)^{n_4}$$

or

$$P(n_1, n_2, n_3, n_4) = (0.13135)(0.33333)^{n_1} (0.53334)^{n_2} (0.16667)^{n_3} (0.49334)^{n_4}$$

The mean number of customers (waiting and in service) in each of the queues Q_1, Q_2,, Q_3 and Q_4, may then be found to be *0.5, 1.14286, 0.2* and *0.973684*, respectively with the mean number of jobs in the overall system as *2.81654*.
The mean time spent in the system by a customer will then be *2.81654/0.1= 28.1654*