Solution to Problem 4.7

The batch service time X^* is either X or $(2X+\Delta)$, each with probability *0.5*. The L.T. of its pdf will then be given by

$$L_{B*}(s) = 0.5L_B(s) + 0.5L_B^2(s)e^{-s\Delta} = 0.5L_B(s)\left[1 + L_B(s)e^{-s\Delta}\right]$$

with moments $\overline{X^*} = 1.5\overline{X} + 0.5\Delta \quad \overline{X^{*2}} = 2.5\overline{X^2} + 2\overline{X}\Delta + 0.5\Delta^2$

Using this, the mean queueing delay W_{qb} before service can start to a batch is

$$W_{qb} = \frac{\lambda\overline{X^{*2}}}{2(1-\rho)} \quad \text{with} \quad \rho = \lambda\overline{X^*} = \lambda\overline{X}\,\overline{r}$$

The mean queueing delay W_2 within a batch will be

$$W_2 = \frac{\overline{r^2} - \overline{r}}{2\overline{r}}\,\overline{X} = \frac{2.5 - 1.5}{2(1.5)}\,\overline{X} = \frac{1}{3}\,\overline{X}$$

$$\left[Alternatively, W_2 = \left(\frac{P\{batch\,size \geq 2\}}{mean\,batch\,size}\right)\overline{X} = \frac{1}{3}\,\overline{X}\right]$$

The mean total queueing delay W_q may then be found as $W_{qb}+W_2$ and will be

$$W_q = \frac{\lambda\left(2.5\overline{X^2} + 2\overline{X}\Delta + 0.5\Delta^2\right)}{2\left(1 - 1.5\lambda\overline{X} - 0.5\lambda\Delta\right)} + \frac{1}{3}\,\overline{X}$$