Solution to Problem 3.3

Since work is conserved, the busy period distribution $L_{BP}(s)$ of this queue will be the same as that for a normal M/G/1 queue and may be obtained by solving Eq. (3.19) as

$$L_{BP}(s) = L_B\left(s + l - lL_{BP}(s)\right)$$

Moreover, the time spent in system by an arriving customer will be equal to the busy period because of the way the queue works. Therefore, $L_{BP}(s)$ will also be the distribution of time spent in system by an arriving customer.

Note that the number of times service will be interrupted for a customer will then be equal to the number of arrivals in the busy period and will be given by $L_{BP}(l - lz)$. Let $F(z)$ be the generating function of the number of interruptions encountered by a customer during its stay in the system. (Note that we define service interruptions as those caused by any other arrival while the original customer is still in the system.)

$$F(z) = L_{BP}(l - lz)$$

$$F'(z) = -lL_{BP}(l - lz) \qquad L'_{BP}(s) = [1 - L_{BP}(s)]L'_B(s + l - lL_{BP}(s))$$

$$\overline{BP} = -L'_{BP}(s)\Big|_{s=0} = (1 + l\,\overline{BP})\overline{X} \quad or \quad \overline{BP} = \frac{\overline{X}}{1 - l\overline{X}}$$

Therefore the mean number of interruptions $= \dfrac{l\overline{X}}{1 - l\overline{X}} = \dfrac{r}{1 - r}$