Solution to Problem 2.9

(a) The state transition diagram may be drawn as given in Fig. 1.2. Note the two different kinds of states when the number in the system lies between K_L and K_H.

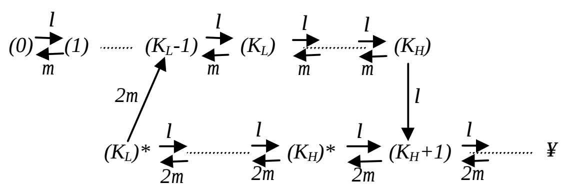

Figure 1.2. State Transition Diagram

(b) Using balance equations, one can then obtain the state probabilities as follows.

$$i = 0,\ldots\ldots\ldots,(K_L - 1) \qquad p_i = r^i p_0$$

$$i = (K_H + 2),\ldots\ldots\ldots\infty \qquad p_i = \left(\frac{r}{2}\right)^{i-(K_H+1)} p_{(K_H+1)}$$

$$i = (K_L + 1),\ldots\ldots,(K_H) \qquad p_i = r^{\,i-(K_L+1)} p_{(K_L+1)}$$

$$i = (K_L + 1)^*,\ldots,(K_H)^* \qquad p_i = \left(\frac{r}{2}\right)^{i-(K_L+1)} p_{(K_L+1)^*}$$

$$p_{(K_L+1)^*} = (1 + \frac{r}{2}) p_{(K_L)^*} \qquad p_{(K_L+1)} = r p_{(K_L)}$$

$$p_{(K_L)} = p_0 \left(\frac{r^{K_L}}{1 + r^{K_H - K_L + 1}} \right)$$

$$p_{(K_H)^*} = p_0 \left(\frac{r}{2}\right)^{K_H - K_L + 1} (1 + \frac{2}{r}) \left(\frac{r^{K_H}}{1 + r^{K_H - K_L + 1}} \right)$$

$$p_{(K_H+1)} = \frac{r}{2} (p_{(K_H)} + p_{(K_H)^*})$$

The usual normalization condition may then be used to get the value of p_0 from which the actual state probabilities may be obtained.