# Solution to Problem 2.20

The state transition diagram for this system will be as shown in Fig. 1.15. The state is represented as (*m*, *j*) where *m* is the number of jobs in the system and *j* is the stage that the job is currently in.

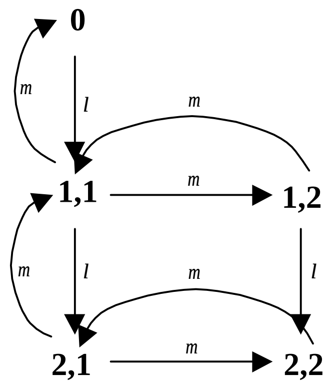


*Figure 1.15.* State Transition Diagram

Choosing convenient closed boundaries, the balance equations for this system may be written as

$$l p_0 = m p_{11} = (l + m) p_{12}$$
$$l (p_{11} + p_{12}) = m p_{21}$$
$$p_{22} + p_{12} = p_{11} + p_{21}$$

which gives

$$p_{11} = r p_0 \quad p_{12} = \frac{r}{1+r} p_0 \quad p_{21} = \frac{r^2 (2+r)}{1+r} p_0 \quad p_{22} = \frac{r^2 (3+r)}{1+r} p_0$$

Applying the normalisation condition to this, we get

$$p_0 = \frac{1+r}{1+3r+6r^2+2r^3}$$

The probability $P_B$ that an arrival leaves without service is ($p_{21}$+$p_{22}$). This leads to

$$P_B = \frac{r^2 (5+2r)}{(1+r)} p_0 \qquad \text{and} \qquad l_{eff} = l (1 - P_B) = l \frac{1+3r+r^2}{1+3r+6r^2+2r^3}$$

Since $N_q = p_{21} + p_{22} = \dfrac{r^2(5+2r)}{1+3r+6r^2+2r^3}$ , we get $W_q = \dfrac{r^2(5+2r)}{l(1+3r+r^2)}$

The Laplace Transform of the effective overall service distribution will be given by

$$L_B(s) = 0.5\left(\frac{2m}{s+2m}\right)\sum_{i=0}^{\infty}\left[0.5\left(\frac{2m}{s+2m}\right)\left(\frac{m}{s+m}\right)\right]^i$$

$$= \frac{m(s+m)}{(s^2+3ms+m^2)}$$