Solution to Problem 2.15

The system's server may be represented by the service facility shown in Fig. 1.9.

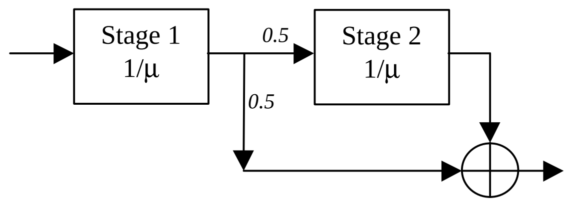

Figure 1.9. Service Facility Model

As usual, we represent the system state by (*n*, *m*) where *n* is the number in the system and *m* is the stage at which the currently served customer may be found. The corresponding state transition diagram is given in Fig. 1.10.

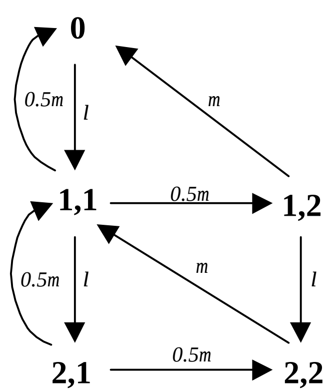

Figure 1.10. State Transition Diagram

The corresponding balance equations are

$$l p_0 = 0.5 m p_{11} + m p_{12} \qquad (l + m) p_{11} = l p_0 + 0.5 m p_{21} + m p_{22}$$
$$(l + m) p_{12} = 0.5 m p_{11} \qquad m p_{21} = l p_{11} \qquad m p_{22} = l p_{12} + 0.5 m p_{21}$$

Choosing any four of these (i.e. the most convenient ones), we can get the state probabilities as

$$p_{11} = p_0 \frac{2r(1+r)}{2+r} \qquad p_{12} = p_0 \frac{r}{2+r}$$

$$p_{21} = p_0 \frac{2r^2(1+r)}{2+r} \qquad p_{22} = p_0 r^2$$

Applying the normalisation condition to these, we get

$$p_0 = \frac{2+r}{2+4r+6r^2+3r^3}$$

The *Average Departure Rate from the Queue* may be found using either of the two approaches -

(a) $0.5m(p_{11}+p_{21})+m(p_{12}+p_{22})$ or (b) $l(p_0+p_{11}+p_{12})$

Using either approach, we get this to be $l\left(2p_0\frac{(1+r)^2}{(2+r)}\right)$