Solution to Problem 2.13

(a) Let n be the number in system when both Prof. Calculus and M. Tintin are working and n^* the number in system when only Prof. Calculus is working.

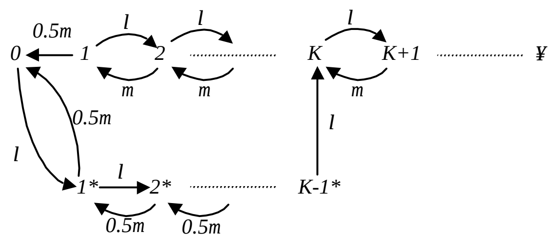

Figure 1.6. State Transition Diagram

Using $r = l/m$

$$p_2 = (p_1 + p_{1*})\, r$$
$$(p_1 + p_{1*}) = 2 p_0 r$$
$$p_n = r^{n-2} p_2 = 2 r^n p_0 \qquad n = 3,4,5......\infty$$

The normalisation condition may then be applied to give

$$p_0 [1 + 2r + 2r^2 + 2r^3 + ...\infty] = 1$$
$$p_0 = \frac{1 - r}{1 + r}$$

The other state probabilities may now be found as

$$p_{1*} = p_0 \frac{2r}{1 + 2r} \qquad p_1 = p_0 \frac{4r^2}{1 + 2r} \qquad p_2 = 2r^2 p_0$$
$$p_n = 2r^n p_0 \qquad n = 3,4,5......\infty$$

Mean number of students in the conference room $= \dfrac{2r}{(1 - r^2)}$

$$\text{P\{Prof. Calculus is working\}} = 1 - p_0 - 0.5p_1 = \frac{2r^2(1+r+r^2)}{(1+r)(1+2r)}$$

$$\text{P\{M. Tintin is working\}} = 0.5p_1 + \sum_{i=2}^{\infty} p_i = \frac{2r^2(2+r)}{(1+r)(1+2r)}$$