

Solution to Problem 2.1

Given that

$$\begin{aligned} \mathbf{I}_i &= (N-i)\mathbf{I} & i = 0,1,2,\dots,N \\ \mathbf{m}_i &= \mathbf{m} & i = 1,2,\dots,N \end{aligned}$$

the flow balance equations may then be written as

$$\begin{aligned} N\mathbf{I}p_0 &= \mathbf{m}p_1 & p_1 &= N\mathbf{r}p_0 & \mathbf{r} &= \frac{\mathbf{I}}{\mathbf{m}} \\ (N-k)\mathbf{I}p_k &= \mathbf{m}p_{k+1} & p_{k+1} &= (N-k)\mathbf{r}p_k \\ \mathbf{I}p_{N-1} &= \mathbf{m}p_N & p_N &= \mathbf{r}p_{N-1} \end{aligned}$$

or
$$p_k = \frac{N!}{(N-k)!} \mathbf{r}^k p_0 \quad \text{for } k=0,1,2,\dots,N$$

The probability p_0 may be found by applying the normalisation condition that $\sum_{k=0}^{\infty} p_k = 1$

$$p_0 \sum_{k=0}^{\infty} \frac{N!}{(N-k)!} \mathbf{r}^k = 1 \quad \text{giving } p_0 = \frac{1}{\sum_{k=0}^{\infty} \frac{N!}{(N-k)!} \mathbf{r}^k}$$