EC 633, Queueing Systems
Home Assignment No. 4
Solutions

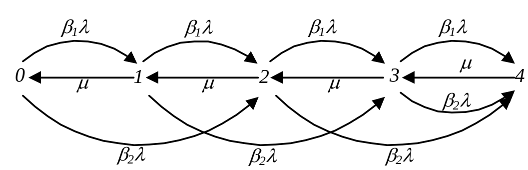
2. We consider separately the case for the PBAS and WBAS approaches

Partial Batch Acceptance Strategy (PBAS)

The state transition diagram for this case is given below. This may be used to write the corresponding balance equations, which may be solved to get the state probabilities as

$$p_0\lambda(\beta_1+\beta_2)=\mu p_1$$
$$p_1\lambda(\beta_1+\beta_2)+p_0\lambda\beta_2=\mu p_2$$
$$p_2\lambda(\beta_1+\beta_2)+p_1\lambda\beta_2=\mu p_3$$
$$p_3\lambda(\beta_1+\beta_2)+p_2\lambda\beta_2=\mu p_4$$

These may be solved to obtain the state probabilities. Note that the normalisation condition is needed to get the value of p_0.



PBAS State Transition Diagram

$$p_1=p_0\phi \qquad \text{where } \rho=\lambda/\mu,\phi=\rho(\beta_1+\beta_2)$$
$$p_2=p_0(\phi^2+\rho\beta_2)$$
$$p_3=p_0(\phi^3+2\phi\rho\beta_2)$$
$$p_4=p_0(\phi^4+3\phi^2\rho\beta_2+(\rho\beta_2)^2)$$
$$\text{with } p_0=\left[1+\phi+\phi^2+\phi^3+\phi^4+(\rho\beta_2)(1+2\phi+3\phi^2)+(\rho\beta_2)^2\right]^{-1}$$

Using the state probabilities as given above, the mean number in the system may be found as

$$N=\sum_{i=1}^{\infty}p_i$$

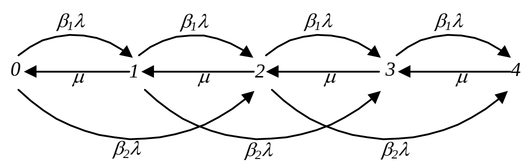
Note that the mean flow rate of jobs offered to the system is $\lambda(\beta_1+2\beta_2)$. Of the total flow rate of jobs offered, the flow rate of jobs refused entry into the system are λ[p_4

$(\beta_1+2\beta_2)$ + $p_3\,\beta_2$]. Therefore, the fraction of jobs refused entry to the system will be given by

$$P_{Blocked\ Jobs} = \frac{p_4(\beta_1 + 2\beta_2) + p_3\beta_2}{(\beta_1 + 2\beta_2)}$$

Whole Batch Acceptance Strategy (WBAS)

The state transition diagram for this case is given below. Note the difference between this and the PBAS case earlier.



WBAS State Transition Diagram

The balance equations for this case are

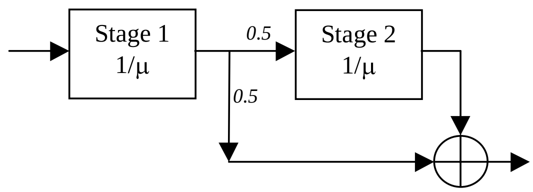
$$\begin{aligned}
&p_0\lambda(\beta_1 + \beta_2) = \mu p_1 \\
&p_1\lambda(\beta_1 + \beta_2) + p_0\lambda\beta_2 = \mu p_2 \\
&p_2\lambda(\beta_1 + \beta_2) + p_1\lambda\beta_2 = \mu p_3 \\
&p_3\lambda\beta_1 + p_2\lambda\beta_2 = \mu p_4
\end{aligned}$$

Solving these, we get

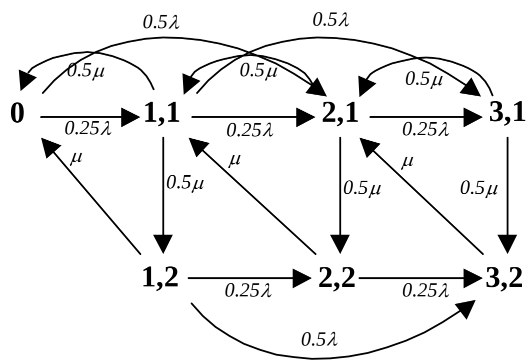
$$\begin{aligned}
&p_1 = p_0\phi \qquad\qquad where\ \ \rho = \lambda/\mu, \phi = \rho(\beta_1 + \beta_2) \\
&p_2 = p_0(\phi^2 + \rho\beta_2) \\
&p_3 = p_0(\phi^3 + 2\phi\rho\beta_2) \\
&p_4 = p_0(\phi^3\rho\beta_1 + \phi^2\rho\beta_2 + 2\phi\rho^2\beta_1\beta_2 + (\rho\beta_2)^2) \\
&with\ p_0 = \begin{bmatrix} 1 + \phi(1 + 2\rho^2\beta_1\beta_2) + \phi^2(1 + \rho\beta_2) + \phi^3(1 + \rho\beta_1) + \\ (\rho\beta_2)(1 + 2\phi) + (\rho\beta_2)^2 \end{bmatrix}^{-1}
\end{aligned}$$

For the WBAS case, the probability that a batch will be refused entry will be [$p_3\beta_2$ + $p_4\,(\beta_1+\beta_2)$]

3 **(a)** The service facility for this case is as given below.



Using this, the state transition diagram for this system may be drawn as shown.



For finding the state probabilities, the following balance equations may be used.

$$0.75\lambda p_0 = 0.5\mu p_{11} + \mu p_{12}$$
$$p_{12} + p_{22} + p_{32} = p_{11} + p_{21} + p_{31}$$
$$p_{12}(0.75\lambda + \mu) = 0.5\mu p_{11}$$
$$p_{22}(0.25\lambda + \mu) = 0.5\mu p_{21} + 0.25\lambda p_{12}$$
$$\mu p_{31} = 0.25\lambda p_{21} + 0.5\lambda p_{11}$$
$$p_{11}(0.75\lambda + \mu) = 0.25\lambda p_0 + \mu p_{22} + 0.5\mu p_{21}$$

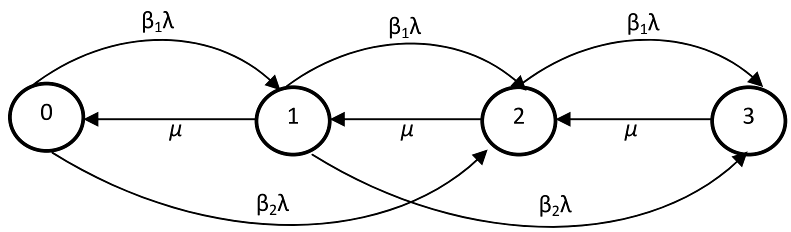
These may be used along with the normalisation condition

$$(p_0 + p_{11} + p_{12} + p_{21} + p_{22} + p_{31} + p_{32}) = 1$$

to get the desired state probabilities.

The probability that a batch is refused entry may then be found as *$0.75(p_{31}+p_{32})+0.5(p_{21}+p_{22})$* using the state probabilities obtained earlier.

(b) In this case, the state transition diagram will be as given below.



The corresponding balance equations are –

$$(\beta_1 + \beta_2)\lambda p_0 = \mu p_1 \qquad \rho = \frac{\lambda}{\mu} \qquad p_1 = (\beta_1 + \beta_2)\rho p_0$$

$$(\beta_1 + \beta_2)\lambda p_1 + \beta_2 \lambda p_0 = \mu p_2 \qquad p_2 = \left((\beta_1 + \beta_2)^2 \rho^2 + \beta_2 \rho\right) p_0$$

$$\beta_1 \lambda p_2 + \beta_2 \lambda p_1 = \mu p_3$$

$$p_3 = (\beta_1 \rho p_2 + \beta_2 \rho p_1) p_0 = \left(\beta_1 (\beta_1 + \beta_2)^2 \rho^3 + \left(2\beta_1\beta_2 + \beta_2{}^2\right)\rho^2\right) p_0$$

The Normalization Condition will then give

$$p_0 = \frac{1}{1 + (\beta_1 + 2\beta_2)\rho + \left(\beta_1^2 + 4\beta_1\beta_2 + 2\beta_2^2\right)\rho^2 + \beta_1(\beta_1 + \beta_2)^2 \rho^3}$$

This can then be used to get the other state probabilities.

The probability that a batch is refused entry into the queue will be

P{Batch refused entry to queue} = $p_2 \beta_2 + p_3(\beta_1 + \beta_2)$