EC 633, Queueing Systems, QUIZ-I (Solution)

Name: **Roll No.:**

1. At time t=0, we find two servers working in the system. The servers provide service with exponentially distributed service times with means μ^{-1} and $(0.5\mu)^{-1}$ respectively for server A and server B. The next departure from this system occurs at t=T. Using the service time distributions of the individual servers, obtain the pdf of T (i.e. $f_T(t)$) and from this show that the overall system works with service rate 1.5μ.

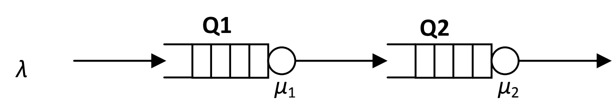
The pdf's of the service times of the two servers A and B are, respectively, $\mu e^{-\mu t}$ and $0.5\mu e^{-0.5\mu t}$ for t≥0. The corresponding cdf's are $1-e^{-\mu t}$ and $1-e^{-0.5\mu t}$ respectively.

Let T be the time to the first departure after t=0 with T_A (T_B) as the time to the first departure from server A (B) after t=0. Then

$$P\{T>t\}=P\{T_A>t,\ T_B>t\}=P\{T_A>t\}P\{T_B>t\}=e^{-\mu t}e^{-0.5\mu t}=e^{-1.5\mu t}$$

The cdf of T would therefore be $1-e^{-1.5\mu t}$. From this we can obtain the pdf of T as $f_T(t)=1.5\mu e^{-1.5\mu t}$ t≥0, which is equivalent to a server working at rate 1.5μ.

2. Consider the system with tandem queues as shown below where arrivals come from a Poisson process with rate λ and Q1 and Q2 provide service at rates μ_1 and μ_2, respectively. The system state is represented by (n_1, n_2) where n_j is the number in Qj, j=1,2

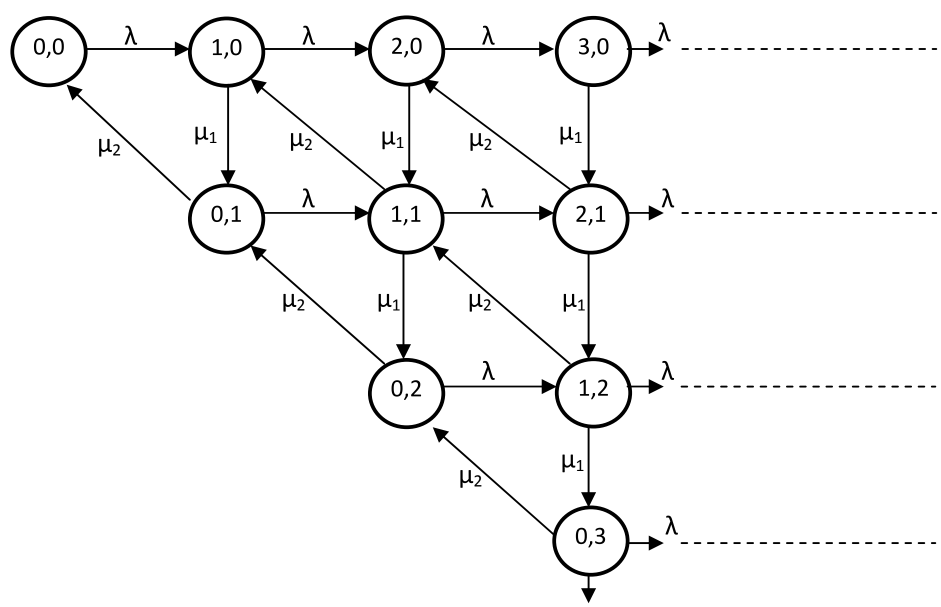


(a) Draw the state transition diagram of the overall system.

(b) Using this state transition diagram, write the global balance equations for states (0,1) (1,1) and (2,0) and show that in each case the following is satisfied -

$p(n_1,n_2)= p(n_1)\, p(n_2)$

Here $p(n_1)$ and $p(n_2)$ are the state probability distributions of Q1 and Q2 considered separately.



We know that $p(n_1) = \rho_1^{n_1}(1-\rho_1)$ n_1=0,1,2,.... and $p(n_2) = \rho_2^{n_2}(1-\rho_2)$ n_2=0,1,2,.... with $\rho_1 = \lambda/\mu_1$ and $\rho_2 = \lambda/\mu_2$. By writing the appropriate balance equations, we want to show that $p(n_1,n_2) = p(n_1)p(n_2) = \rho_1^{n_1}\rho_2^{n_2}(1-\rho_1)(1-\rho_2)$ satisfies the global balance equations for the designated states and would therefore be a valid state distribution for the tandem queueing system.

(a) State (0,1) $p_{0,1}(\lambda+\mu_2) = p_{1,0}\mu_1 + p_{0,2}\mu_2 \quad \Rightarrow \quad p_{0,1}(\rho_2+1) = p_{1,0}\frac{\mu_1}{\mu_2} + p_{0,2}$

LHS= $\rho_2(1-\rho_1)(1-\rho_2)(\rho_2+1) = (1-\rho_1)(1-\rho_2)(\rho_2^2+\rho_2)$

RHS= $\rho_1(1-\rho_1)(1-\rho_2)\frac{\rho_2}{\rho_1} + \rho_2^2(1-\rho_1)(1-\rho_2) = (1-\rho_1)(1-\rho_2)(\rho_2^2+\rho_2)$ =LHS

(b) State (1,1) $p_{1,1}(\lambda+\mu_1+\mu_2) = p_{2,0}\mu_1 + p_{0,1}\lambda + p_{1,2}\mu_2$

$$\Rightarrow \quad p_{1,1}\left(\rho_1 + 1 + \frac{\mu_2}{\mu_1}\right) = p_{2,0} + p_{0,1}\rho_1 + p_{1,2}\frac{\mu_2}{\mu_1}$$

LHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^2\rho_2 + \rho_1\rho_2 + \rho_1^2\right)$

RHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^2 + \rho_1\rho_2 + \rho_1\rho_2^2\frac{\rho_1}{\rho_2}\right)$ =LHS

(c) State (2,0) $p_{2,0}(\lambda+\mu_1) = p_{1,0}\lambda + p_{2,1}\mu_2 \quad \Rightarrow \quad p_{2,0}(\rho_1+1) = p_{1,0}\rho_1 + p_{2,1}\frac{\rho_1}{\rho_2}$

LHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^3 + \rho_1^2\right)$

RHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^2 + \rho_1^3\right)$=LHS

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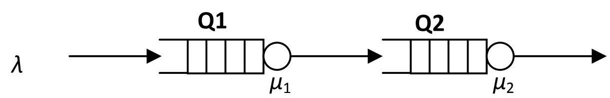
The pdf's of the service times of the two servers A and B are, respectively, $\mu e^{-\mu t}$ and $2\mu e^{-2\mu t}$ for t≥0. The corresponding cdf's are $1-e^{-\mu t}$ and $1-e^{-2\mu t}$ respectively.

Let T be the time to the first departure after t=0 with T_A (T_B) as the time to the first departure from server A (B) after t=0. Then

$$P\{T>t\}=P\{T_A>t, T_B>t\}=P\{T_A>t\}P\{T_B>t\}=e^{-\mu t}e^{-2\mu t}=e^{-3\mu t}$$

The cdf of T would therefore be $1-e^{-3\mu t}$. From this we can obtain the pdf of T as $f_T(t)=3\mu e^{-3\mu t}$ t≥0, which is equivalent to a server working at rate 3μ.

2. Consider the system with tandem queues as shown below where arrivals come from a Poisson process with rate λ and Q1 and Q2 provide service at rates μ_1 and μ_2, respectively. The system state is represented by (n_1, n_2) where n_j is the number in Qj, j=1,2

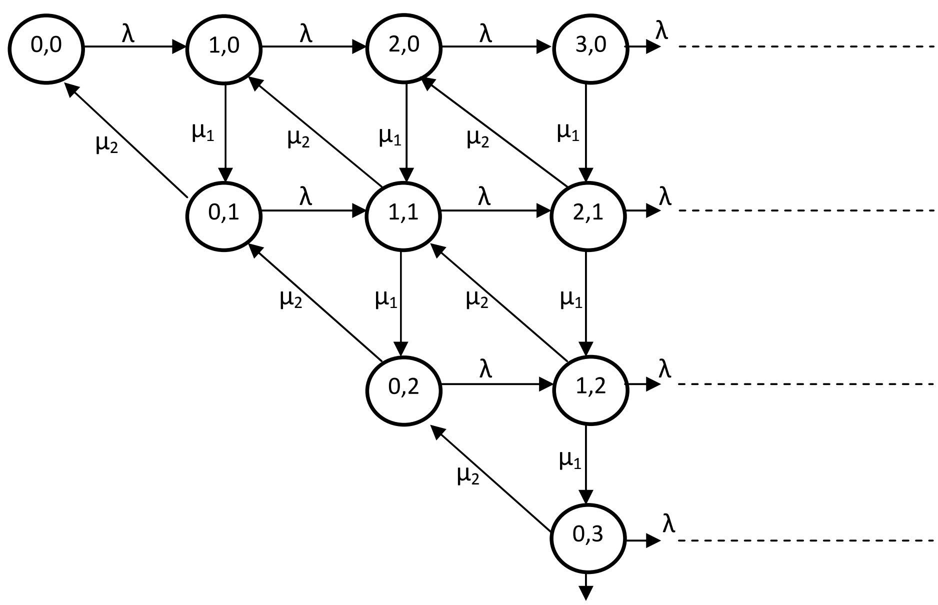


(a) Draw the state transition diagram of the overall system.

(b) Using this state transition diagram, write the global balance equations for states (1,0) (1,1) and (0,2) and show that in each case the following is satisfied –

$p(n_1,n_2)= p(n_1)\, p(n_2)$

Here $p(n_1)$ and $p(n_2)$ are the state probability distributions of Q1 and Q2 considered separately.



We know that $p(n_1) = \rho_1^{n_1}(1-\rho_1)$ n_1=0,1,2,.... and $p(n_2) = \rho_2^{n_2}(1-\rho_2)$ n_2=0,1,2,.... with $\rho_1 = \lambda/\mu_1$ and $\rho_2 = \lambda/\mu_2$. By writing the appropriate balance equations, we want to show that $p(n_1,n_2) = p(n_1)p(n_2) = \rho_1^{n_1}\rho_2^{n_2}(1-\rho_1)(1-\rho_2)$ satisfies the global balance equations for the designated states and would therefore be a valid state distribution for the tandem queueing system.

(a) State (1,0) $p_{1,0}(\lambda+\mu_1) = p_{0,0}\lambda + p_{1,1}\mu_2 \quad \Rightarrow \quad p_{1,0}(\rho_1+1) = p_{1,1}\dfrac{\mu_2}{\mu_1} + p_{0,0}\rho_1$

LHS= $\rho_1(1-\rho_1)(1-\rho_2)(\rho_1+1) = (1-\rho_1)(1-\rho_2)(\rho_1^2+\rho_1)$

RHS= $\rho_1\rho_2(1-\rho_1)(1-\rho_2)\dfrac{\rho_1}{\rho_2} + (1-\rho_1)(1-\rho_2)\rho_1 = (1-\rho_1)(1-\rho_2)(\rho_1^2+\rho_1)$ =LHS

(b) State (1,1) $p_{1,1}(\lambda+\mu_1+\mu_2) = p_{2,0}\mu_1 + p_{0,1}\lambda + p_{1,2}\mu_2$

$$\Rightarrow \quad p_{1,1}\left(\rho_1 + 1 + \frac{\mu_2}{\mu_1}\right) = p_{2,0} + p_{0,1}\rho_1 + p_{1,2}\frac{\mu_2}{\mu_1}$$

LHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^2\rho_2 + \rho_1\rho_2 + \rho_1^2\right)$

RHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1^2 + \rho_1\rho_2 + \rho_1\rho_2^2\dfrac{\rho_1}{\rho_2}\right)$ =LHS

(c) State (0,2) $p_{0,2}(\lambda+\mu_2) = p_{1,1}\mu_1 + p_{0,3}\mu_2 \quad \Rightarrow \quad p_{0,2}(\rho_2+1) = p_{1,1}\dfrac{\mu_1}{\mu_2} + p_{0,3}$

LHS= $(1-\rho_1)(1-\rho_2)\left(\rho_2^3 + \rho_2^2\right)$

RHS= $(1-\rho_1)(1-\rho_2)\left(\rho_1\rho_2\dfrac{\rho_2}{\rho_1} + \rho_2^3\right) = (1-\rho_1)(1-\rho_2)\left(\rho_2^2+\rho_2^3\right)$=LHS