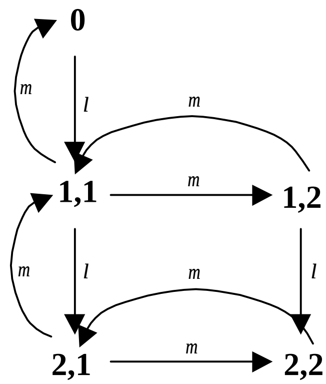
EE 679, Queueing Systems (2001-02F)
Solutions to Test -3

1. The state transition diagram for this system will be as shown below. The state is represented as (*m*, *j*) where *m* is the number of jobs in the system and *j* is the stage that the job is currently in.



State Transition Diagram

Choosing convenient closed boundaries, the balance equations for this system may be written as

$$l p_0 = m p_{11} = (l + m) p_{12}$$
$$l (p_{11} + p_{12}) = m p_{21}$$
$$p_{22} + p_{12} = p_{11} + p_{21}$$

which gives

$$p_{11} = r p_0 \quad p_{12} = \frac{r}{1+r} p_0 \quad p_{21} = \frac{r^2(2+r)}{1+r} p_0 \quad p_{22} = \frac{r^2(3+r)}{1+r} p_0$$

Applying the normalisation condition to this, we get

$$p_0 = \frac{1+r}{1+3r+6r^2+2r^3}$$

The probability P_B that an arrival leaves without service is (p_{21}+p_{22}). This leads to

$$P_B = \frac{r^2(5+2r)}{(1+r)} p_0 \qquad \text{and} \qquad l_{eff} = l(1-P_B) = l \frac{1+3r+r^2}{1+3r+6r^2+2r^3}$$

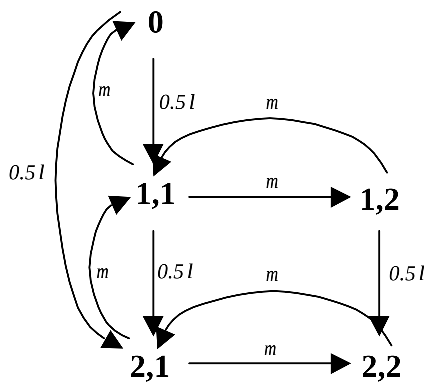
Since $N_q = p_{21} + p_{22} = \dfrac{r^2(5+2r)}{1+3r+6r^2+2r^3}$, we get $W_q = \dfrac{r^2(5+2r)}{l(1+3r+r^2)}$

The Laplace Transform of the effective overall service distribution will be given by

$$L_B(s) = 0.5\left(\frac{2m}{s+2m}\right)\sum_{i=0}^{\infty}\left[0.5\left(\frac{2m}{s+2m}\right)\left(\frac{m}{s+m}\right)\right]^i$$
$$= \frac{m(s+m)}{(s^2+3ms+m^2)}$$

2. This is the same server model as the one considered in Problem 1, except that arrivals now come from a batch process where a batch will have either one or two jobs with probability 0.5 each.

With this modification, the state transition diagram of Problem 1 may be modified to give the new state transition diagram shown below.



State Transition Diagram

The balance equations will then be

$$lp_0 = mp_{11} = (0.5l + m)p_{12}$$
$$0.5l(p_{11} + p_{12} + p_0) = mp_{21}$$
$$p_{22} + p_{12} = p_{11} + p_{21}$$

which gives

$$p_{11} = rp_0 \qquad p_{12} = \frac{2r}{2+r}p_0$$
$$p_{21} = \frac{r(2+5r+r^2)}{2(2+r)}p_0 \qquad p_{22} = \frac{r(2+7r+r^2)}{2(2+r)}p_0$$

Applying the normalization condition to this, we get

$$p_0 = \frac{2 + r}{2 + 7r + 7r^2 + r^2}$$

In this case, the total job flow offered will be $0.5l+(0.5)(2l)=1.5l$

Of this, the total flow blocked will be $[(p_{11}+p_{12})(0.5)(2l)+(p_{21}+p_{22})\{(0.5)l+(0.5)(2l)\}]=l[p_{11}+p_{12}+1.5(p_{21}+p_{22})]$.

Therefore the ratio of blocked jobs will be $[p_{11}+p_{12}+1.5(p_{21}+p_{22})]/1.5$.