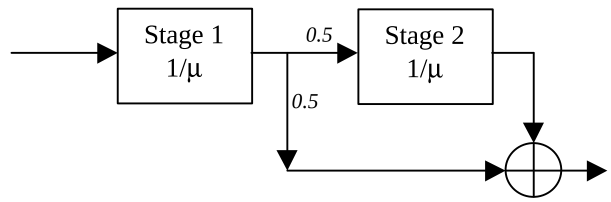
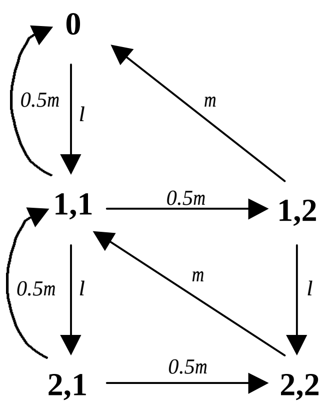
EE 679, Queueing Systems (2000-01F)
Solutions to Test -3

1. For the service time distribution given, the service may be considered as being given in two stages as shown in the figure.



Let System State = *(n, m)* where *n* is the number in the system and *m* the the stage where the customer currently being served is to be found. The state transition may be drawn as shown. The balance equations for these can also be written as follows -



$$l p_0 = 0.5 m p_{11} + m p_{12}$$
$$(l + m) p_{11} = l p_0 + 0.5 m p_{21} + m p_{22}$$
$$(l + m) p_{12} = 0.5 m p_{11}$$
$$m p_{21} = l p_{11}$$
$$m p_{22} = l p_{12} + 0.5 m p_{21}$$

Choosing any four of these (i.e. the most convenient ones), we solve for the state probabilities in terms of p_0 to get -

$$p_{11} = p_0 \frac{2r(1+r)}{2+r} \qquad p_{12} = p_0 \frac{r}{2+r} \qquad p_{21} = p_0 \frac{2r^2(1+r)}{2+r} \qquad p_{22} = p_0 r^2$$

Now using the Normalizing Condition, we can find p_0 to be -

$$p_0 = \frac{2+r}{2+4r+6r^2+3r^3}$$

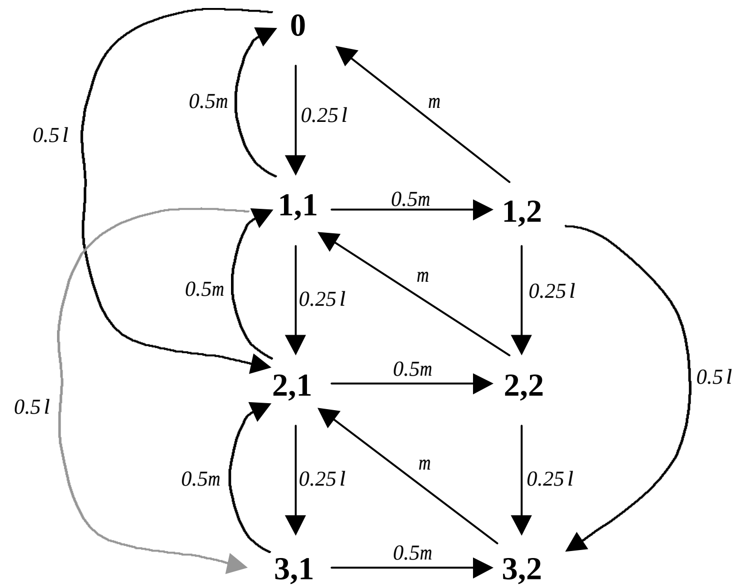
The *Average Departure Rate from the Queue* may be found using either of the two approaches -

(a) $0.5m(p_{11} + p_{21}) + m(p_{12} + p_{22})$

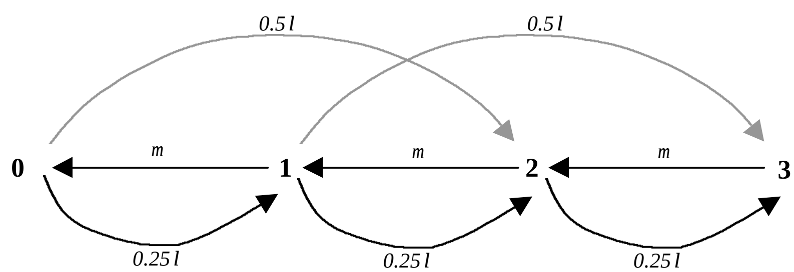
or (b) $l(p_0 + p_{11} + p_{12})$

Using either approach, we get this to be $l\left(2p_0 \frac{(1+r)^2}{(2+r)}\right)$

2. (a)



2. (b)



$$mp_1 = (0.75l)p_0$$

$$mp_2 = (m + 0.75l)p_1 - (0.25l)p_0 \qquad \textit{Balance Equations}$$

& $$mp_3 = (0.25l)p_2 + (0.5l)p_1$$

Solving, we get -

$$p_1 = (0.75r)p_0$$

$$p_2 = (0.5r + 0.5625r^2)p_0$$

$$p_3 = (0.5r^2 + 0.140625r^3)p_0$$

with $p_0 = \dfrac{1}{1 + 1.25r + 1.0625r^2 + 0.140625r^3}$ from *normalizing conditions*

P{batch is refused entry into the queue}

$$= p_2(0.5) + p_3(0.75)$$

$$= (0.25r + 0.65625r^2 + 0.10546875r^3)p_0$$