EE 679, Queueing Systems (2001-02F)
Solutions to Test -2

1. Applying Eqs. (2.7) and (2.8) and using $r = \frac{l}{m}$, we can directly get that

$$p_k = p_0 \prod_{i=0}^{k-1} \frac{l_i}{m_{i+1}} = p_0 (k+1) r^k$$

Using the normalization condition, we can then obtain

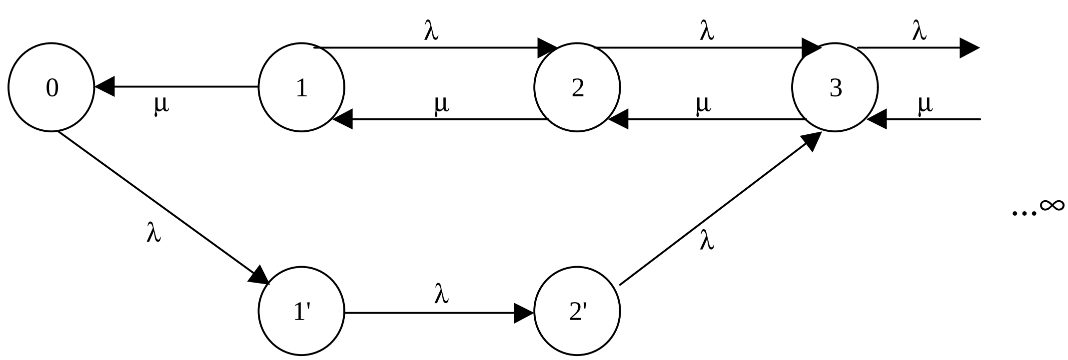
$$p_0 = (1-r)^2$$
$$p_k = (k+1)(1-r)^2 r^k \quad k = 0,1,......\infty$$

The mean number, *N*, in the system will be given by

$$N = \sum_{k=0}^{\infty} k p_k = \frac{2r}{(1-r)}$$

We can see that the series summation required for applying the normalization condition $\sum_{k=0}^{\infty} p_k = 1$ may be evaluated only if $r<1$ or $l<m$. This is the condition required for the queue to be stable.

2. The state transition for this system diagram is given below.



The balance equations may be written as follows.

$$l p_0 = l p_{1'} = l p_{2'}$$
$$l p_0 = m p_1$$
$$(l + m) p_1 = m p_2$$
$$(l + m) p_2 = l p_1 + m p_3$$
$$p_{n+1} m = l p_n \qquad n = 3,4,\ldots\ldots\ldots\ldots\infty$$

These may be solved to get the following state probabilities.

$$p_{2'} = p_{1'} = p_0$$
$$p_1 = r p_0$$
$$p_2 = r(1 + r) p_0$$
$$p_3 = r(1 + r + r^2) p_0$$
$$p_n = r^{n-3} p_3 \qquad n = 4,5,\ldots\ldots\ldots\infty$$

where application of the normalization condition yields $p_0 = \dfrac{1 - r}{3}$

Let P{*k*}=P{*k* users in the system}. Using the state probabilities given above, we can find these as

$$P\{1\} = p_1 + p_{1'} = p_0(1 + r)$$
$$P\{2\} = p_2 + p_{2'} = p_0(1 + r + r^2)$$
$$P\{3\} = p_3 = p_0(1 + r + r^2) r$$
$$P\{n\} = p_0(1 + r + r^2) r^{n-2} \qquad n = 2,3,4,\ldots\ldots\ldots\infty$$

The server in this system will be busy in all states other than states *0, 1', and 2'*. Therefore, the probability $P_{server\ busy}$ will be given by

$$P_{server\ busy} = 1 - p_0 - p_{1'} - p_{2'} = 1 - 3p_0 = r$$

The interesting thing to note is that even in spite of its different behavior, the server has to work just as hard it would in a normal M/M/1 queue.