EE 679, Queueing Systems (2001-02F)
Solutions to Test -1

1. The balance equations for this system may be written as follows

$$lp_0 = mp_1$$
$$lp_i = mp_{i+1} \quad i = 1,.......,(K-1)$$

which gives the state probabilities as $p_i = r^i p_0 \quad i = 1,.....,K$. Using the normalization condition, we will get the actual state probabilities as

$$p_i = \frac{(1-r)r^i}{1-r^{K+1}} \quad i = 0,1,.......,K$$

The mean number N in the system will be

$$N = \sum_{k=0}^{K} kp_k = \frac{r\left[1-(1+K)r^K + Kr^{K+1}\right]}{(1-r^{K+1})(1-r)}$$

and the probability that an arrival will leave without service will be $p_K = \dfrac{r^K(1-r)}{(1-r^{K+1})}$

Note that since this is a finite buffer queue, it would always be stable.

2. Consider the balance equations for the states *1'*, *2'* .... etc.. These are

$$lp_{1'} = 0.5\,lp_0 \quad lp_{2'} = 0.5\,lp_{1'} \,..........\, lp_{n'} = 0.5\,lp_{(n-1)'} \,..........$$

giving $p_{n'} = (0.5)^n p_0 \quad n = 1,2,........\infty$

Note that $\sum_{n=1}^{\infty} p_{n'} = 0.5\,p_0 \dfrac{1}{1-0.5} = p_0$

Now consider the balance equations for the others states. These are

$$lp_0 = mp_1 \quad lp_1 + 0.5\,lp_{1'} = mp_2 \quad .... \quad lp_{n-1} + 0.5\,lp_{(n-1)'} = mp_n \quad ....$$

Using $r = \dfrac{l}{m}$, we get that

$$p_1 = rp_0$$
$$p_n = rp_{n-1} + 0.5\,rp_{(n-1)'} \quad n > 1$$

We can also observe that $\sum_{n=1}^{\infty} p_n = \frac{1.5r}{1-r}\, p_0$

Applying the normalization condition yields

$$2p_0 + \frac{1.5r}{1-r}\, p_0 = 1 \qquad \text{or} \qquad p_0 = \frac{1-r}{2-0.5r}$$

which may be used to find the individual state probabilities using the expressions given earlier.