EE633 2014-2015F
Quiz – II Solutions

Time: 45 minutes *Marks 10*

1. (Using standard notation as given in the lectures)

$p_{d,00} = \alpha_0 = L_B(\lambda) \qquad p_{d,01} = 1-\alpha_0 = 1-L_B(\lambda)$

$p_{d,10} = \alpha_0 = L_B(\lambda) \qquad p_{d,11} = 1-\alpha_0 = 1-L_B(\lambda)$

Therefore, the balance equation is $p_{d,0}[1-L_B(\lambda)] = p_{d,1}L_B(\lambda)$

Normalization Condition $p_{d,0} + p_{d,1} = 1$

Therefore, $p_{d,0} = L_B(\lambda) \quad p_{d,1} = 1-L_B(\lambda)$

Let P_B be the blocking probability. Then $\rho_c = \lambda \overline{X}(1-P_B) = \rho(1-P_B)$

Then $p_0 = 1-\rho_c = 1-\rho(1-P_B)$ and also $p_0 = (1-P_B)p_{d,0} = (1-P_B)L_B(\lambda)$

Solving, we get $P_B = \dfrac{L_B(\lambda)+\rho-1}{L_B(\lambda)+\rho}$

Therefore, the required equilibrium state probabilities as seen at any time instant are –

$$p_2 = P_B = \frac{L_B(\lambda)+\rho-1}{L_B(\lambda)+\rho}$$

$$p_1 = (1-P_B)p_{d,1} = \frac{1-L_B(\lambda)}{L_B(\lambda)+\rho}$$

$$p_0 = (1-P_B)p_{d,0} = \frac{L_B(\lambda)}{L_B(\lambda)+\rho}$$

2. The effective service time distribution $L_B^*(s)$ will be –

$$L_B^*(s) = \sum_{k=1}^{\infty}(1-p)L_B(s)[pL_B(s)]^{k-1} = \frac{(1-p)L_B(s)}{1-pL_B(s)}$$

with mean effective service time as $\overline{X^*} = \dfrac{\overline{X}}{1-p}$

Using the results of Problem 1, we can then obtain the following –

$$p_2 = P_B = \frac{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)} + \dfrac{\rho}{1-p} - 1}{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)} + \dfrac{\rho}{1-p}} \qquad p_0 = \frac{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)}}{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)} + \dfrac{\rho}{1-p}}$$

$$p_1 = \frac{1-\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)}}{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)} + \dfrac{\rho}{1-p}} = \frac{\dfrac{1-L_B(\lambda)}{1-pL_B(\lambda)}}{\dfrac{(1-p)L_B(\lambda)}{1-pL_B(\lambda)} + \dfrac{\rho}{1-p}}$$