EE 633

Quiz -II

Maximum Marks 10

1. Mean number of jobs in a batch = 0.25 +0.25(2)+0.5(3) = 2.25

P{randomly selected job is first job of a batch} = $\frac{1}{2.25} = 0.444$

P{randomly selected job is second job of a batch} = $\frac{0.75}{2.25} = 0.333$

P{randomly selected job is first job of a batch} = $\frac{0.5}{2.25} = 0.222$

Mean Batch Service Time= $\bar{X} = \alpha(1) + 0.75\beta(1) + 0.5\gamma(1)$ $\qquad\qquad \rho = \lambda\bar{X}$

$$\begin{aligned}\overline{X^2} &= 0.25\overline{X_1^2} + 0.25\overline{(X_1+X_2)^2} + 0.5\overline{(X_1+X_2+X_3)^2} \\ &= \overline{X_1^2} + 0.75\overline{X_2^2} + 0.5\overline{X_3^2} + 1.5\overline{X_1}\,\overline{X_2} + \overline{X_2}\,\overline{X_3} + \overline{X_1}\,\overline{X_3} \\ &= \alpha(2) + 0.75\beta(2) + 0.5\gamma(2) + 1.5\alpha(1)\beta(1) + \beta(1)\gamma(1) + \alpha(1)\gamma(1)\end{aligned}$$

$$\begin{aligned}L_B(s) &= 0.25L_\alpha(s) + 0.25L_\alpha(s)L_\beta(s) + 0.5L_\alpha(s)L_\beta(s)L_\gamma(s) \\ &= 0.25L_\alpha(s)\left[1 + L_\beta(s) + 2L_\beta(s)L_\gamma(s)\right]\end{aligned}$$

Therefore, using ρ, $\overline{X^2}$ and $L_B(s)$ from above, for a batch considered as one "job", we get

$$L_{Wqb}(s) = \frac{s(1-\rho)}{s - \lambda + \lambda L_B(s)}$$

$$W_{qb} = \frac{\lambda\overline{X^2}}{2(1-\rho)}$$

Using these, for an arbitrarily chosen job, we will get

$$\begin{aligned}W_q &= W_{qb} + \frac{0.75}{2.25}\alpha(1) + \frac{0.5}{2.25}\left[\alpha(1) + \beta(1)\right] \\ &= W_{qb} + \frac{1.25}{2.25}\alpha(1) + \frac{0.5}{2.25}\beta(1)\end{aligned}$$

and $$L_{Wq}(s) = L_{Wqb}(s)\left[\frac{1}{2.25} + \frac{0.75}{2.25}L_\alpha(s) + \frac{0.5}{2.25}L_\alpha(s)L_\beta(s)\right]$$

2. $0.5(0.25\lambda_2 + 0.5\lambda_1) = \lambda_2 = \lambda_3 \quad \Rightarrow \quad \lambda_2 = \lambda_3 = 0.286\lambda_1 \quad \lambda_4 = 2\lambda + 0.25\lambda_2 = 2\lambda + 0.071\lambda_1$

$\lambda_1 = \lambda + \lambda_3 + \lambda_4 = 3\lambda + 0.357\lambda_1 \quad \Rightarrow \quad \lambda_1 = 4.666\lambda$

Therefore, $\tilde{\lambda} = (4.666\lambda, 1.334\lambda, 1.334\lambda, 2.331\lambda)$ *Sanity Check*= $\lambda_X + \lambda_Y = 3\lambda$

(a) For the network to be stable, we require $4.666\lambda < \mu$ or $\rho < 0.214$

(b) $\tilde{\lambda} = (0.9332, 0.2668, 0.2668, 0.4662)$

$\tilde{\rho} = (0.9332, 0.2668, 0.2668, 0.4662)$

$\tilde{N} = (13.97, 0.364, 0.364, 0.873)$

$\tilde{W} = (14.97, 1.364, 1.364, 1.873)$

Average number of jobs in the system = 15.571

Mean Transit Time from A or B and leaving from X or Y =15.571/(0.6) = 25.952

To calculate the Mean Transit Time for jobs entering from A and leaving from X or Y, set λ_B=0

Then $0.5(0.25\lambda_2 + 0.5\lambda_1) = \lambda_2 = \lambda_3 \quad \Rightarrow \quad \lambda_2 = \lambda_3 = 0.286\lambda_1 \quad \lambda_4 = 0.25\lambda_2 = 0.071\lambda_1$

$\lambda_1 = \lambda + \lambda_3 + \lambda_4 = \lambda + 0.357\lambda_1 \quad \Rightarrow \quad \lambda_1 = 1.555\lambda = 0.311$

$\tilde{\lambda} = (0.311, 0.089, 0.089, 0.022)$

Visit Ratios $\tilde{V}^* = (1.555, 0.445, 0.445, 0.11)$

Mean Transit Time from A and leaving from X or Y

$= 1.555 * 14.97 + 0.445 * 1.364 + 0.445 * 1.364 + 0.11 * 1.873$

= 24.698