

EE633 2014-2015F

Quiz – I

Time: 45 minutes

Marks 10

Note: Questions and parts of questions can be done in any order you wish

1. Circular on Usage of VIP Toilets (as issued by the *Competent Toilet Authority*)

Henceforth, the VIP toilet in the office will be made available for non-VIPs subject to the following rules.

- i. No non-VIP user will be allowed to wait if the toilet is currently occupied by a VIP or non-VIP user
- ii. If a VIP user arrives while a non-VIP user is using the toilet, the non-VIP user will be **immediately** evicted and the VIP user will be allowed to use the toilet
- iii. VIP users can wait (infinite waiting space available) if another VIP user is currently using the toilet

You have been deputed by the Office Audit Team to study the effectiveness of these rules when the system is operating at equilibrium. You have determined that VIP users arrive from a Poisson process with average arrival rate λ_V and occupy the toilet for an exponentially distributed time with mean $1/\mu_V$. Non-VIP users arrive from a Poisson process with average arrival rate λ_N and occupy the toilet for an exponentially distributed time with mean $1/\mu_N$. (For Notational Convenience, $\rho_V = \frac{\lambda_V}{\mu_V}$, $\rho_N = \frac{\lambda_N}{\mu_N}$)

Denote the **system state** by {Number of VIP users in the system, Number of Non-VIP users in the system}

Find the following -

- a) What is the probability that a VIP user will have to wait to use the toilet? **[1]**
- b) Draw the State Transition Diagram for the system **[2]**
- c) Write the Balance Equations and solve them to obtain the state probabilities **[4]**
- d) What is the overall efficiency of toilet usage? (i.e. fraction of time toilet is in use) **[0.5]**
- e) What is ratio of toilet usage of non-VIP customers to that of VIP customers **[0.5]**

2. Due to the criticism levelled on the scheme of **(1)**, the *Competent Toilet Authority* has asked you for advice on modifying the scheme to make it fairer for the non-VIP users while retaining some of the VIP privileges. You have advised that the following changes be made to the scheme of (1) –

- i. Non-VIP users are now allowed to wait for the toilet **if** there are no VIP users in the system.
- ii. If a non-VIP user is using the toilet when a VIP customer arrives, the non-VIP user will be allowed to complete his/her toilet usage (i.e. will not be forced to leave immediately!). However, all other waiting non-VIP users will be forced to leave immediately. New non-VIP users will also not be allowed into the toilet system until all VIP users have left.
- iii. VIP users can wait (infinite buffer space) if the toilet is in use by a VIP or non-VIP user

Draw the State Transition Diagram for this modified system

[2]