## Maximum Marks 10

Consider a $M / M / 1$ queue where the server goes for a single vacation of time $T$ whenever the system goes empty. On return from this vacation, the server starts serving whoever may have arrived during the vacation or stays available for the next arrival. Assume that the arrivals to the queue come at rate $\lambda$ and the mean service time is $\bar{X}=\frac{1}{\mu}$. Define $\rho=\lambda \bar{X}=\frac{\lambda}{\mu}$.
(a) If $T$ is a fixed, calculate the mean server idle period and the mean busy period in a cycle and use these to show that the probability of the server being idle is $(1-\rho)$
[1+1+1]
(b) Do the following assuming $\boldsymbol{T}$ to be exponentially distributed with mean $\frac{1}{\beta}$
(i) Draw the State Transition Diagram with an appropriate definition of the system state
(Note: You do not have to write/solve the balance equations unless you need/want to)
(ii) Find the probability $P(V)$ that the server is on vacation
(iii) Show that the probability $P(W)$ that the server is working is $\rho$, as usual.

