EC633 Quiz-2 (01-APR-2011)
Solutions

1.

$$\lambda_1 = \lambda + 0.5\lambda_2 + 0.4\lambda_1 \quad \Rightarrow \quad 0.6\lambda_1 - 0.5\lambda_2 = \lambda$$
$$\lambda_2 = 2\lambda + 0.2\lambda_1 \quad \Rightarrow \quad -0.2\lambda_1 + \quad \lambda_2 = 2\lambda$$

$$\lambda_1 = 4\lambda, \lambda_2 = 2.8\lambda$$
$$\rho_1 = \frac{\lambda_1}{2\mu} = 2\rho \quad \rho_2 = 2.8\rho$$

(a) For stability $\rho_1, \rho_2 < 1$. Therefore $\rho < \min\{0.5, 0.357\} = 0.357$

(b) For ρ=0.3, we have ρ_1=0.6 and ρ_2=0.84.

Therefore, $N_1 = \frac{\rho_1}{1-\rho_1} = \frac{0.6}{0.4} = 1.5 \qquad N_2 = \frac{\rho_2}{1-\rho_2} = \frac{0.84}{0.16} = 5.25$

Since $N = N_1 + N_2 = 6.75$ we have, $W = \frac{N}{\lambda + 2\lambda} = \frac{2.25}{\lambda}$

(c) Note that with inputs applied to both A and B, the average delays in Q1 and Q2 will

$W_1 = \frac{1.5}{4\lambda} = \frac{0.375}{\lambda}$ and $W_2 = \frac{5.25}{2.8\lambda} = \frac{1.875}{\lambda}$

Consider the system where arrivals are only allowed to enter from B (i.e. no external arrivals are allowed from A). We then have –

$$\lambda_1 = 0.5\lambda_2 + 0.4\lambda_1 \qquad 0.6\lambda_1 = 0.5\lambda_2 \qquad \lambda_2 = 1.2\lambda_1$$
$$\lambda_2 = 2\lambda + 0.2\lambda_1 \qquad 1.2\lambda_1 = 2\lambda + 0.2\lambda_1$$

$$\lambda_1 = 2\lambda \quad \lambda_2 = 2.4\lambda$$

For the flow entering from B, the visit ratios for Q1 and Q2 will be -

$$V_{B1} = \frac{\lambda_1}{2\lambda} = 1 \quad V_{B2} = \frac{\lambda_2}{2\lambda} = 1.2$$

Using this, the delay encountered by a job entering the system from B, will be –

$$W_B = V_{B1}W_1 + V_{B2}W_2 = \frac{0.375 + 2.25}{\lambda} = \frac{2.625}{\lambda}$$

[Check: For flow entering only from A, we have –

$$\lambda_1 = \lambda + 0.5\lambda_2 + 0.4\lambda_1 \quad \Rightarrow \quad 0.6\lambda_1 - 0.5\lambda_2 = \lambda$$
$$\lambda_2 = 0.2\lambda_1$$

$$\lambda_1 = 2\lambda \quad \lambda_2 = 0.4\lambda$$

$$V_{A1} = 2 \quad V_{A2} = 0.4$$
$$W_A = \frac{0.75 + 0.75}{\lambda} = \frac{1.5}{\lambda}$$

Then $W = \frac{2W_B + W_A}{3} = \frac{2.25}{\lambda}$

2. We consider, jobs for each priority class separately, starting with the highest priority (Class 3) and ending with the lowest priority (Class 1)

Class 3: Residual Service Time seen by a Class 3 arrival $R_3 = \frac{1}{2}\lambda_3 \overline{X_3^2}$

$$W_{q3} = \frac{R_3}{1-\rho_3} \quad \rho_3 = \lambda_3 \overline{X_3}$$

$$W_3 = W_{q3} + \overline{X_3}$$

Class 2: Residual Service Time seen by a Class 2 arrival $R_2 = \frac{1}{2}\left(\lambda_3 \overline{X_3^2} + \lambda_2 \overline{X_2^2} + \lambda_1 \overline{X_1^2}\right)$

$$W_2 = \overline{X_2} + \frac{R_2}{(1-\rho_2-\rho_3)} + \overline{X_3}\lambda_3 W_2 \quad \Rightarrow \quad W_2 = \frac{\overline{X_2}}{(1-\rho_3)} + \frac{R_2}{(1-\rho_3)(1-\rho_2-\rho_3)}$$

Class 1: Residual Service Time seen by a Class 1 arrival $R_1 = R_2 = \frac{1}{2}\left(\lambda_3 \overline{X_3^2} + \lambda_2 \overline{X_2^2} + \lambda_1 \overline{X_1^2}\right)$

$$W_1 = \overline{X_1} + \frac{R_1}{(1-\rho_1-\rho_2-\rho_3)} + \overline{X_3}\lambda_3 W_1 + \overline{X_2}\lambda_2 W_1$$

$$W_1 = \frac{\overline{X_1}}{(1-\rho_2-\rho_3)} + \frac{R_1}{(1-\rho_2-\rho_3)(1-\rho_1-\rho_2-\rho_3)}$$