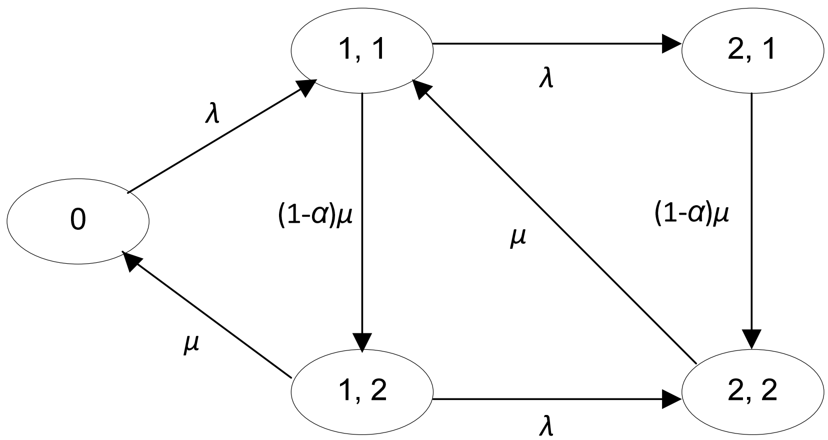
EC633 Quiz-1 Solutions

(a) As usual, we represent the system state as (*n*, *j*) where *n* is the number in the system and *j* is the stage at which the customer currently in service is being served. The state when the system is empty is represented by (0). The corresponding State Transition diagram is given below. **[4]**



(b) The balance equations will be –

$$\lambda p_0 = \mu p_{12} \qquad p_{12} = \rho p_0$$

$$(\lambda + \mu) p_{12} = (1-\alpha)\mu p_{11} \qquad p_{11} = \frac{\rho(1+\rho)}{(1-\alpha)} p_0$$

$$\lambda p_{11} = (1-\alpha)\mu p_{21} \qquad p_{21} = \frac{\rho^2(1+\rho)}{(1-\alpha)^2} p_0$$

$$\mu p_{22} = \lambda p_{12} + (1-\alpha)\mu p_{21}$$

$$\Rightarrow\; p_{22} = \rho^2 p_0 + \frac{\rho^2(1+\rho)}{(1-\alpha)} p_0 \qquad p_{22} = \rho^2 p_0 \left[\frac{2-\alpha+\rho}{(1-\alpha)}\right]$$

Using the normalization condition, we get –

$$p_0 = \frac{1}{\left[1+\rho+\frac{\rho(1+\rho)}{(1-\alpha)}+\frac{\rho^2(1+\rho)}{(1-\alpha)^2}+\rho^2+\frac{\rho^2(1+\rho)}{(1-\alpha)}\right]}$$

Therefore, $p_1 = p_{11}+p_{12} = p_0\rho\left[1+\frac{(1+\rho)}{(1-\alpha)}\right] = p_0\rho\left(\frac{2-\alpha+\rho}{1-\alpha}\right)$ **[8]**

$$p_2 = p_{21} + p_{22} = \frac{\rho^2 p_0}{(1-\alpha)^2}\left[1+\rho+(1-\alpha)(2-\alpha+\rho)\right]$$

$$= \frac{\rho^2 p_0}{(1-\alpha)^2}\left[3+2\rho-3\alpha-\rho\alpha+\alpha^2\right]$$

(c) Let $L_B(s)$ be the L.T. of the overall service time. Then, we can see that -

$$L_B(s) = \left(\frac{\mu}{s+\mu}\right)\left[\alpha L_B(s) + (1-\alpha)\left(\frac{\mu}{s+\mu}\right)\right]$$

[4]

$$L_B(s)\left[1 - \frac{\alpha\mu}{s+\mu}\right] = (1-\alpha)\left(\frac{\mu}{s+\mu}\right)^2 \qquad L_B(s) = \frac{(1-\alpha)\mu^2}{(s+\mu)(s+\mu-\alpha\mu)}$$

(d) Let $\overline{B}$ be the mean service time as offered by this facility. Then, we can see that –

$$\overline{B} = \frac{1}{\mu} + (1-\alpha)\frac{1}{\mu} + \alpha\overline{B}$$

$$\overline{B}(1-\alpha) = \frac{2-\alpha}{\mu} \qquad \overline{B} = \frac{1}{\mu}\left(\frac{2-\alpha}{1-\alpha}\right)$$

If the queue had infinite buffers, then the condition for equilibrium will be –

$$\lambda\overline{B} < 1 \qquad \Rightarrow \quad \rho\left(\frac{2-\alpha}{1-\alpha}\right) < 1$$

or $\alpha < \dfrac{1-2\rho}{1-\rho}$

Since $0 \le \alpha \le 1$ is required (α is a probability), ρ must be less than 0.5 **[4]**