



Consider an $M/1/2$ queue where the service is provided in stages as shown in the figure. Each stage provides service with an exponentially distributed service time with mean $1/\mu$ and the service times of the two stages are independent of each other. The routing probabilities are as shown in the figure. Assume that arrivals come to this queue at rate λ . Define $\rho = \lambda/\mu$ for notational convenience.

- With an appropriately defined system state, draw the state transition diagram for the system. [4]
- Obtain the probability p_j , $j=0,1,2$ of there being j users in the system. [8]
- Find the PDF of the service time (or its L.T.). [4]
- Suppose the queue had infinite buffers and you are allowed to adjust α so as to keep the system stable (i.e. one where it will be in equilibrium). What is the maximum value of ρ for which you can force the system to be in equilibrium by a proper choice of α ? [4]