EE 633, Queueing Systems (2017-18F)
Solution to Quiz – II

(a) If there are k arrivals in the Class 1 service time, then $T = X_1 + k$ Class 2 Busy Periods

$$\overline{T} = \overline{X_1} + (\lambda_2 \overline{X_1})\left(\frac{\overline{X_2}}{1-\lambda_2\overline{X_2}}\right) = \frac{\overline{X_1}}{1-\lambda_2\overline{X_2}} \qquad \textbf{[10]}$$

and $$L_T(s) = \int_{x=0}^{\infty} e^{-sx}\left\{\sum_{k=0}^{\infty}\frac{(\lambda_2 x)^k}{k!}e^{-\lambda_2 x}L_{BP2}^k(s)\right\}b_1(x)dx$$ where $$L_{BP2}(s) = L_{B2}\left(s+\lambda_2-\lambda_2 L_{BP2}(s)\right)$$

Therefore, $$L_T(s) = \int_{x=0}^{\infty} e^{-(s+\lambda_2)x}e^{\lambda_2 x L_{BP2}(s)}b_1(x)dx = L_{B1}\left(s+\lambda_2-\lambda_2 L_{BP2}(s)\right) \qquad \textbf{[30]}$$

(b) **Case I:** No Class 1 arrival in Class 2 Busy Period (Probability $L_{BP2}(\lambda_1)$)

Mean length of BP for this case = Mean of a Class 2 Busy Period = $\dfrac{\overline{X_2}}{1-\lambda_2\overline{X_2}}$ **[10]**

Case II: One or more Class 1 arrivals in Class 2 Busy Period (Probability $\left(1-L_{BP2}(\lambda_1)\right)$

For this case, the Busy Period will be BP2+T where T is as given in (a)

$$\text{Mean length of BP for this case} = \frac{\overline{X_2}}{1-\lambda_2\overline{X_2}} + \frac{\overline{X_1}}{1-\lambda_2\overline{X_2}} = \frac{\overline{X_1}+\overline{X_2}}{1-\lambda_2\overline{X_2}} \qquad \textbf{[20]}$$

Mean length of BP when started by a Class 2 arrival

$$= L_{BP2}(\lambda_1)\left(\frac{\overline{X_2}}{1-\lambda_2\overline{X_2}}\right) + \left(1-L_{BP2}(\lambda_1)\right)\left(\frac{\overline{X_1}+\overline{X_2}}{1-\lambda_2\overline{X_2}}\right) = \frac{\left(\overline{X_1}+\overline{X_2}\right)-\overline{X_1}L_{BP2}(\lambda_1)}{1-\lambda_2\overline{X_2}} \qquad \textbf{[10]}$$

(c) **Mean Length of BP**

$$= \frac{\lambda_1}{\lambda_1+\lambda_2}\left(\frac{\overline{X_1}}{1-\lambda_2\overline{X_2}}\right) + \frac{\lambda_2}{\lambda_1+\lambda_2}\left(\frac{\overline{X_1}+\overline{X_2}-\overline{X_1}L_{BP2}(\lambda_1)}{1-\lambda_2\overline{X_2}}\right)$$

$$= \frac{\left(\lambda_1-\lambda_2 L_{BP2}(\lambda_1)\right)\overline{X_1}+\lambda_2\left(\overline{X_1}+\overline{X_2}\right)}{\left(\lambda_1+\lambda_2\right)\left(1-\lambda_2\overline{X_2}\right)} \qquad \textbf{[20]}$$