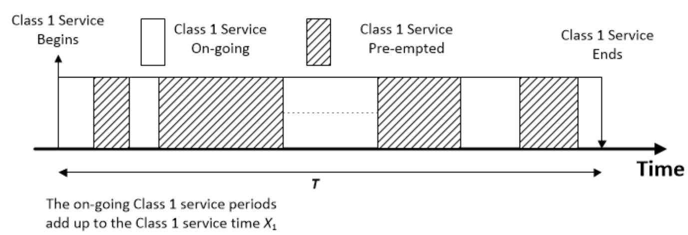
EE 633, Queueing Systems (2017-18F)
Quiz – II

Maximum Marks=100 (will be scaled to 10 later) *Time = 60 minutes*

Answers can be left in terms of appropriate transforms but you must clearly state how those transforms are to be found.

Consider an M/G/1 queue where there are two classes of customers, Class 1 and Class 2 with respective arrival rates λ_1 and λ_2. Entry of the lower priority (Class 1) customers is restricted such that there can **at the most be ONE Class 1 customer in the system at any time** – waiting for service or being served. Class 1 customers denied entry into the system leave without service. Moreover, a Class 1 service can be preempted by Class 2 customers. The pre-emption of Class 1 service, if it occurs, is of the *Preemptive Resume* type. (Of course, a Class 1 service cannot begin if there are any Class 2 customers in the system.) Class 2 customers get served with higher priority and there is no limit on the number of Class 2 customers that can be there in the system. The service time of Class 1 customers is given by the random variable X_1 with the usual parameters $b_1(t), B_1(t), L_{B1}(s)$. The service time of Class 2 customers is given by the random variable X_2 with the usual parameters $b_2(t), B_2(t), L_{B2}(s)$.

(a) Let T be the total (random) duration of a Class 1 service. Find the mean $\overline{T}$ and the Laplace Transform $L_T(s)$ of its pdf **[10+30]**



The system can be viewed as having a Busy-Idle cycle where the Idle Period is when the system is completely empty and the Busy Period may be started by either a Class 1 or Class 2 arrival with respective probabilities $\frac{\lambda_1}{\lambda_1+\lambda_2}$ and $\frac{\lambda_2}{\lambda_1+\lambda_2}$.

(b) What will be the **Mean Length of a Busy Period which is started with a Class 2 arrival**?
[*Hint:* There are two cases to consider here – one where there are no Class 1 arrivals in the initial Class 2 Busy Period and the other where there are one or more Class 1 arrivals in that busy period. The probabilities of these two cases also have to be used.) **[10+20+10]**

(c) What will be the **Mean Length of the Busy Period?** **[20]**

Standard M/G/1 results that may be useful (standard notation)

$$\rho = \lambda\overline{X} \quad P(z) = \frac{(1-\rho)(1-z)L_B(\lambda-\lambda z)}{L_B(\lambda-\lambda z)-z} \quad \overline{BP} = \frac{\overline{X}}{1-\lambda\overline{X}} \quad L_{BP}(s) = L_B\left(s+\lambda-\lambda L_{BP}(s)\right)$$

P{no arrivals in a random interval of length T} = $L_T(\lambda)$

Things to do at home (not graded) – Find the total traffic carried and the Class 1 traffic that is actually carried. Subtracting will give you the Class 2 traffic that is carried. Verify that this is $\lambda_2\overline{X_2}$ as expected!