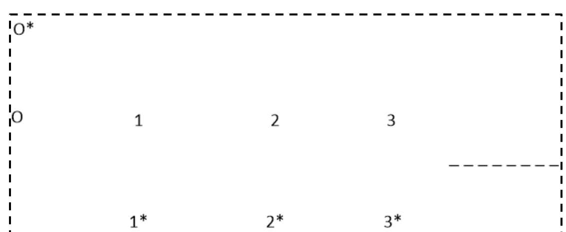
EE 633, Queueing Systems (2017-18F)
Quiz – I

Maximum Marks=100 (will be scaled to 10 later) *Time = 60 minutes*
There will be no part marking other than what is shown, except possibly for (e)
Take your time to read the problem and work carefully

Consider an M/M/1 queue where the server goes for only **ONE** vacation every time it becomes idle (i.e. at the end of each Busy Period). The Vacation Period is exponentially distributed with mean $\frac{1}{\beta}$ and arrival to the queue come from a Poisson process with rate λ. The service times are exponentially distributed with mean $\frac{1}{\mu}$. On returning from the vacation, if the server finds one or more customers in the queue then it starts serving them. On the other hand, if the server returns from its vacation and finds the system still empty, it stays in the system waiting to serve the next customer who arrives.

System State Definitions to be used for (d) and (e) below		
0: server on vacation	0*: server idle but not on vacation	
n: n customers in the system, server working normally		n=1, 2, 3, ……
n*: n customers in system, server on vacation		n=1, 2, 3, ….

- **(a)** What is the **mean idle time** in a cycle? (Idle time is the time when the *server is not working*.) **[5]**
- **(b)** What is the **mean length of the Busy Period** in a cycle? **[10]**
- **(c)** What is the **Mean Cycle Time**? What is the **probability that the server is idle**? What is the **probability that the server is on vacation**? **[5+5+5]**
- **(d)** Draw the **State Transition Diagram** of the system. *Use the system state definitions given above and in the figure* **[10]**



- **(e)** Defining $P(z)=\sum_{n=1}^{\infty} z^n p_n$ and $P^*(z)=\sum_{n=1}^{\infty} z^n p_{n*}$, obtain $P^*(z)$ and derive an expression for $P(z)$ in terms of $P^*(z)$. (Remember to use the *Normalization Condition* to obtain whatever state probability term appears in these expressions.) **[50]**
- **(f)** Use **(e)** to confirm your probability results for **(c).** **[5+5]**