EE633 Queueing Systems (2015-16F)
Quiz – 2

Maximum Marks: 10 Time: (5 + 45) minutes

Please spend 5 minutes to read and understand the question!
It is recommended that the parts be done in sequence.

Consider a **2-priority, pre-emptive resume M/G/1** queue where Class 2 has higher priority than Class 1. The queue restricts the **maximum number of Class 2 jobs in the system to two (2)**, i.e. one in service and at most one more in the buffer. However, it can buffer an **infinite number of Class 1 customers**.

Customers of Class *j* arrive from a Poisson process at rate λ_j and require service given by the random variable X_j $j=1,2$. The service times have mean $\overline{X_j}$, second moment $\overline{X_j^2}$, pdf $b_j(x)$ and L.T. of the pdf as $L_{Bj}(s)$ for $j=1,2$

(a) What are the equilibrium probabilities of finding *n*, *n*=0, 1, 2, Class 2 customers in the system?
(*Hint: Note that Class 2 customers have preemptive resume priority over the Class 1 customers but that at most two Class 2 customers can be in the system!*) **[2]**

(b) What would be the condition on $\rho_1 = \lambda_1 \overline{X_1}$ for the system to be stable for Class 1 customers? **[1]**

(c) Find the mean and distribution (pdf or L.T. of pdf) of the busy period of the queue for Class 2 customers **[1+2]**

(d) A Class 1 customer starts service at time *t*=0 and finally finishes its service at time *T*. Find the mean and distribution (pdf or L.T. of pdf) of the random variable *T*. **[1+2]**
(*Hint: The random variable T would be the effective service time of a Class 1 customer which would consist of its actual service times X_1 and the periods for when the Class 1 service is preempted. Note also that whenever there is a Class 2 arrival while the Class 1 customer is being served, a Class 2 busy period starts whose length would be the length of that particular preemption period.*)

(e) What would be the **equilibrium state distribution of Class 1 customers** in this system? **[1]**
(*Hint:* $P(z) = \dfrac{(1-\rho)(1-z)A(z)}{A(z)-z}$ *for the standard M/G/1 queue with proper definitions of the individual terms.*)