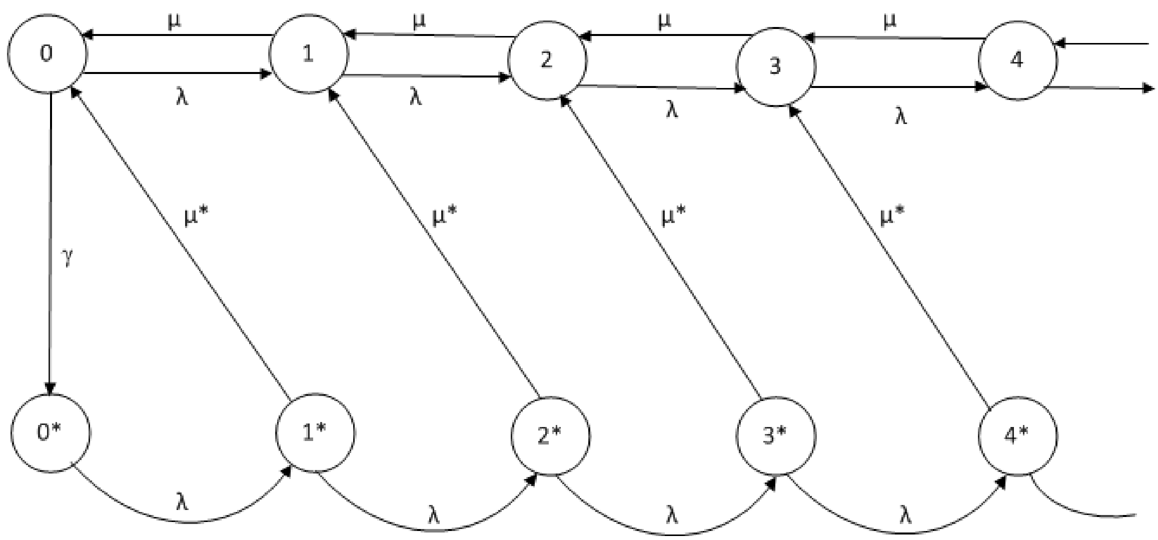
Quiz – I (EE633 Queueing Systems) 2015-16F
Solution

(a) State Transition Diagram



(b) Balance Equations

(i) $p_0\gamma = p_0^*\lambda \quad \Rightarrow \quad p_0^* = \frac{\gamma}{\lambda}p_0$

(ii)

$$p_1^*(\lambda+\mu^*) = p_0^*\lambda$$
$$p_2^*(\lambda+\mu^*) = p_1^*\lambda$$
$$p_3^*(\lambda+\mu^*) = p_2^*\lambda$$

....

....

(iii)

$$\lambda(p_0 + p_0^*) = \mu p_1 + \mu^* p_1^*$$
$$\lambda(p_1 + p_1^*) = \mu p_2 + \mu^* p_2^*$$
$$\lambda(p_2 + p_2^*) = \mu p_3 + \mu^* p_3^*$$

....

....

(c) Generating Functions

From (ii), we get $\quad (\lambda+\mu^*)\left[P^*(z) - p_0^*\right] = \lambda z P^*(z)$

Therefore, $\quad P^*(z) = p_0^*\frac{(\lambda+\mu^*)}{(\lambda+\mu^*-\lambda z)} = p_0\frac{\gamma(\lambda+\mu^*)}{\lambda(\lambda+\mu^*-\lambda z)}$ **[Equation 1]**

From (iii), we get $\quad \lambda z\left[P(z)+P^*(z)\right] = \mu\left[P(z)-p_0\right] + \mu^*\left[P^*(z)-p_0^*\right]$

$$(\mu-\lambda z)P(z) + (\mu^*-\lambda z)P^*(z) = \mu p_0 + \mu^* p_0^*$$
$$= p_0\left[\mu + \frac{\mu^*\gamma}{\lambda}\right]$$

[Equation 2]

(d) Probability of System being empty = $p_0 + p_0^*$

In this case, the Normalization Condition is that $P(1) + P^*(1) = 1$

From Eq (1), $P^*(1) = p_0 \dfrac{\gamma(\lambda + \mu^*)}{\lambda\mu^*}$

From Eq (2) $(\mu - \lambda)P(1) + (\mu^* - \lambda)P^*(1) = p_0 \left[\mu + \dfrac{\mu^*\gamma}{\lambda} \right]$

$$\left.\begin{aligned} (\mu - \lambda)P(1) + p_0 \frac{\gamma(\mu^{*2} - \lambda^2)}{\lambda\mu^*} &= p_0 \left[\mu + \frac{\mu^*\gamma}{\lambda} \right] \\ (\mu - \lambda)P(1) &= p_0 \left[\mu + \frac{\gamma\lambda}{\mu^*} \right] \end{aligned}\right\} \Rightarrow P(1) = p_0 \frac{\mu\mu^* + \lambda\gamma}{\mu^*(\mu - \lambda)}$$

Therefore, $p_0 \left[\dfrac{\mu\mu^* + \lambda\gamma}{\mu^*(\mu - \lambda)} + \dfrac{\gamma(\lambda + \mu^*)}{\lambda\mu^*} \right] = 1$

$$p_0 \left[\frac{\lambda\mu\mu^* + \lambda^2\gamma + \gamma(\mu^* + \lambda)(\mu - \lambda)}{\lambda\mu^*(\mu - \lambda)} \right] = 1$$

$$p_0 = \frac{\lambda\mu^*(\mu - \lambda)}{\lambda\mu\mu^* + \gamma\mu\mu^* - \gamma\mu^*\lambda + \gamma\lambda\mu} = \frac{\lambda\mu^*(\mu - \lambda)}{\mu\mu^*(\lambda + \gamma) + \lambda\gamma(\mu - \mu^*)}$$

P{system is empty}= $p_0 + p_0^* = p_0 \dfrac{(\lambda + \gamma)}{\lambda}$

$$= \frac{\mu^*(\mu - \lambda)(\lambda + \gamma)}{\mu\mu^*(\lambda + \gamma) + \lambda\gamma(\mu - \mu^*)}$$