EE633 Queueing Systems (2015-16F)
Quiz – I

Maximum Marks: 10 | Time: (5 + 45) minutes

Please spend 5 minutes to read and understand the question!

Modified Exceptional First Service (M/M/1/∞):

Consider a single server queue where arrivals come from a Poisson process with rate λ. Normal service is given at rate μ with exponentially distributed service times. When the system becomes empty, it first enters a **state "0"** where if an arrival comes then normal service at rate μ is resumed once again. However, if the system is in state "0" and there are no arrivals for a *sufficiently long time* (**defined later**), then the system enters **state "0*".** The first arrival coming when the system is in this state is served at rate μ^* (also with exponentially distributed service time); other subsequent arrivals coming before the system becomes empty once again get normal service at rate μ as before.

Assume that the **transition rate from state "0" to state "0*" is given by** γ, i.e. the time spent in state "0" when there are no arrivals is exponentially distributed with mean γ^{-1}.

Define the system state as $\{n\}$ n=1,2, …… when the system is working with rate μ and as $\{n^*\}$ n=1,2,3,… when the system is working with rate μ^* with state probabilities p_n and p_n^*, respectively. Define the corresponding generating functions as $P(z) = \sum_{n=0}^{\infty} p_n z^n$ and $P^*(z) = \sum_{n=0}^{\infty} p_n^* z^n$

(a) Draw the State Transition Diagram for this system **[2]**
(b) Write the balance equations for this system **[2]**
(c) Obtain two equations that can be used to solve for $P(z)$ and $P^*(z)$ in terms of p_0 (or, in terms of p_0^*) and λ, μ, μ^* and γ **[2]**
Note: You are not being asked to explicitly solve for $P(z)$ and $P^*(z)$
(d)Using the equations obtained in part (c), find the overall probability of the system being empty, i.e. $p_0 + p_0^*$, in terms of λ, μ, μ^* and γ. **[4]**

Hint to check your answer in part (d) – Think of what will happen to this queue if either $\gamma = 0$ or $\mu^* = \mu$ or both! Does your answer reduce to what you would expect in these cases?