EC633 Mid-Term Examination

Maximum Marks=60 *Time=2 hours*

Attempt all questions. Show all relevant steps. One A4/Letter sheet of notes allowed.

1. Consider an M/M/1/∞ queue where the server works at rate μ when the system starts. Subsequently, whenever the number in the system (waiting + in service) reaches 2, the server increases its service rate to 2μ. Once it starts serving at this higher rate 2μ, it continues to serve at that rate until the system becomes empty – when that happens it reverts back to a service rate of μ. This process continues as long as the system is in operation, i.e. it again goes to the higher rate when there are two users in the system and reverts to the lower rate when the system becomes empty. Arrivals to the queue come at rate λ with $\rho=\lambda/\mu$.

(a) Draw the state transition diagram for the system. **[5]**

(b) Obtain the system state probabilities and use these to get the probabilities of finding *j* users in the system for *j*=0, 1, 2 ... **[10]**

[*The normalization result can be left as a summation but the terms of the summation must be explicitly stated*.]

(c) What condition must be satisfied for the system to be in equilibrium? Give reasons for your answer **[5]**

2. Consider the M/G/1 queue where the server goes for a **single** vacation of random length *V* every time it becomes idle. In addition, the first service time in every busy period requires an extra service time Δ (fixed) over and above the usual random service time X. (In the following, use standard notation, i.e. $\lambda, X, \overline{X}, \overline{X^2}, L_B(s), V, \overline{V}, \overline{V^2}, L_V(s)$ as appropriate.)

(a) What are the mean length of the idle period and the mean length of the busy period? **[2+4]**

(b) From (a), find the probability that the server is busy. **[2]**

(c) Use (b) to find the mean service time for a job in this system? **[2]**

3. The L.T. of the pdf of the busy period of an M/G/1 queue is $L_{BP}(s) = L_B(s + \lambda - \lambda L_{BP}(s))$ where λ is the average arrival rate of jobs to the queue. The service time has mean $\overline{X}$ and second moment $\overline{X^2}$ with $L_B(s)$ as the L.T. of its pdf. Derive the first two moments, i.e. $\overline{BP}$ and $\overline{BP^2}$, of the busy period. **[5+5]**

4. Consider an M/G/1 queue, where once the system becomes empty, service starts again only after ***K*** customers have arrived. Assume that arrivals come to this queue at rate λ and the mean service time is $\overline{X}$ with offered traffic as $\rho = \lambda \overline{X}$. (In the following, use standard notation, i.e. $X, \overline{X}, \overline{X^2}, b(x), B(x), L_B(s), A(z)$ etc. as appropriate.)

(a) Verify that even for this queue, P{server is idle}=1-ρ **[5]**

(b) Analyze this queue using the **Imbedded Markov Chain** approach to obtain *P*(*z*) the generating function for the number in the system. **[10]**

(c) Use (b) to calculate the mean number in the system. **[5]**