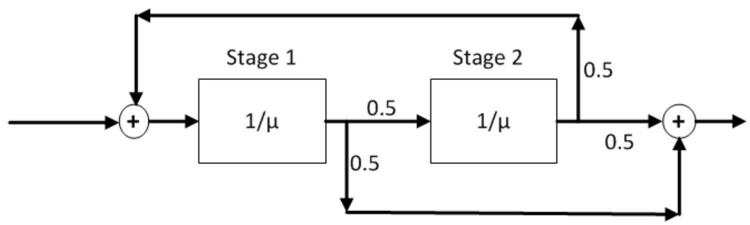
EE633 Queueing Systems (2015-16F)
Mid-Term Examination

Maximum Marks: 30 Time: 120 minutes

Please spend 5 minutes to read and understand the questions! The marks indicated are a good estimator of the amount of effort that may be required.
Please read carefully the instructions for Question 2
*This question paper has **TWO** problems and **TWO** pages*

1. The server of an M/-/1 queue can be modeled as shown in the figure where each stage provides an exponentially distributed service time with mean $1/\mu$. Arrivals to this queue come from a Poisson process at rate λ. **Use** $\rho = \frac{\lambda}{\mu}$ **for notational convenience**



Server Model

(a) Give the pdf (or its Laplace Transform) of the overall service time of this server **[2]**
(b) What is the mean service time? **[1]**
(c) What is the condition for this queue to be stable (i.e. in equilibrium)? **[1]**
(d) Using the standard definition of state for the Method of Stages, draw the **State Transition Diagram** of the queue. **[2]**
[**State Definition**: We define the state as(n, j) where n is the number in the system and j is the stage of the server in which the customer is currently being served.]
(e) What is the probability of finding the server idle? **[1]**
(f) Define the generating functions to be $P_1(z) = \sum_{n=1}^{\infty} p_{n,1} z^n$ and $P_2(z) = \sum_{n=1}^{\infty} p_{n,2} z^n$. Find these generating functions in terms of ρ . **[4]**
[*Note: Be careful to state and use the Normalization Condition appropriately, if needed*.]
[g] What is the *generating function for the number in the system*? **[1]**
[h] What is the *Mean Number in the System*? **[1]**
[i] What is the second moment of the service time for this server, i.e. $\overline{X^2}$ where X is the random service time provided by this server? [*Hint: Think creatively if you want to reduce your work!*] **[2]**

2. Modified Exceptional First Service for M/G/1 Queue *Please note that you can EITHER do the full question for the full 15 marks OR do the simplified question for 7 marks. YOU MUST INDICATE CLEARLY (when you start your answer) WHICH CHOICE YOU ARE EXERCISING. If you try both then you MUST cross out the one that you do not want graded! If you do not do this and I find both solutions in your answer script then I will only grade the FULL QUESTION and disregard whatever you may have written for the other.*

Full Question (15 Marks) Consider an M/G/1 queue where arrivals come from a Poisson process with rate λ. The normal service times have a general distribution with L.T. of its pdf given by $L_B(s)$ where the mean and second moment are $\overline{X}$ and $\overline{X^2}$, respectively. Whenever the idle time of the server exceeds T (fixed), the first service starting the corresponding busy period has L.T. of it pdf as $L_B^*(s)$ with mean and second moment $\overline{X^*}$ and $\overline{X^{*2}}$,respectively; all other service times are normal as given before.

For notational convenience, you may use $\overline{X^*} = \overline{X} + \overline{\Delta}$ and $\overline{X^{*2}} = \overline{X^2} + 2(\overline{X})(\overline{\Delta}) + \overline{\Delta^2}$, $A(z) = L_B(\lambda - \lambda z)$ and $A^*(z) = L_B^*(\lambda - \lambda z)$, if required.

(*Hint: P{idle period longer than T}=e*$^{-\lambda T}$ *This is then also the probability that a busy period will start with an exceptional first-service. Otherwise, the busy period will start with a normal service.*}

(a) Find the mean waiting time in queue W_q for this queue using the *Residual Life Approach*. **[7]**

(b) Use the *Imbedded Markov Chain Approach* to analyze this queue and obtain the generating function $P(z)$ for the number in the system. **[8]**

(*Note: Your answer is incomplete if you do not find* p_0)

Simplified Question (7 marks): Same as above except that we set $T = 0$, i.e., the first service time of a busy period is always exceptional. **Do parts (a) and (b) for this simplified version**. **[3+4]**