EE633 2014-2015F

Mid Term Exam

Time: 2 hours *Marks 30*

Note: (a) The mark allotted to each part is indicative of the effort required to do that part.

(b) Probability distributions can be found as generating functions or Laplace Transforms of the PDF

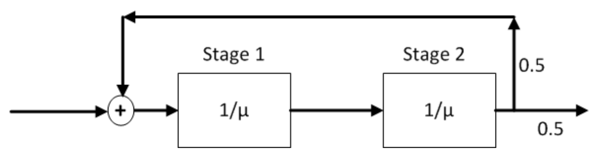
1. Consider an M/M/2 queue with two servers A and B, with respective service rates μ_A and μ_B, where the service times are exponentially distributed. If an arrival finds the system empty then it gets served by the server which had been serving last, just before the system became idle. In all other situations, the system behaves like a normal M/M/2 queue, i.e. customers are served in a FCFS fashion by whichever server becomes available. Assume arrivals come at rate λ.

(a) Draw a state transition diagram for the system with an appropriate definition of the system states. **[3]**

(b) Obtain *closed form expressions* for the system state probabilities **[5]**

(c) The servers are penalized in proportion to the time for which they are idle. What is the **ratio** in which servers A and B would be penalized? **[2]**

2. A M/-/1 queue has a service facility with the server providing service in two stages as shown, where each stage provides service which has an exponential distribution with mean $1/\mu$. Customers come for service with mean rate λ.



As per normal practice, let (n, j) denote the state of the queue where n is the stage at which the customer is currently being served and j is the number in the system. Let "0" represent the state when the system is empty. For notational convenience, let $\rho = \lambda / \mu$

Define the generating functions $P_1(z)$ and $P_2(z)$ as $P_1(z) \triangleq \sum_{n=1}^{\infty} p_{1,n} z^n$ and $P_2(z) \triangleq \sum_{n=1}^{\infty} p_{2,n} z^n$

(a) What is the mean service time of the server and the probability of the system being empty? **[0.5+0.5]**

(b) Draw the **State Transition Diagram** of the system. **[2]**

(c) Obtain *closed form expressions* for $p_{1,1}, p_{1,2}, p_{2,1}$ and $p_{2,2}$ **as a function of** ρ **[4]**

(d) Express $P_2(z)$ in terms of **only** z, ρ and $P_1(z)$

(e) Obtain another expression relating $P_1(z)$ and $P_2(z)$ which can be used with the relation given in (e) and the result of (c) to find $P_1(z)$ and $P_2(z)$. (*Note*: *Just give the expressions! You do not need to solve them to get the actual* $P_1(z)$ *and* $P_2(z)$) **[for both (d) and (e), Marks =3]**

3. A single server queue at equilibrium has mean arrival rate λ from a Poisson process. The normal service time is exponentially distributed with mean $\overline{X} = 1/\mu$. The first service after the system becomes idle requires an additional (**fixed**) service time Δ.

(a) Derive the mean length of the Busy Period for this queue **[1]**

(b) What is the probability that the system is observed to be empty? **[1]**

(c) Obtain the state probability distribution **[5]**

(d) Use the result of (c) to find the mean number in the system **[3]**