EE 633, Queueing Systems
Mid-Term Examination (2012-2013S)

Max Marks Obtainable 30 **(Total Marks 40)**

1. Consider an **M/M/1/2** queue which serves two classes of jobs, LP and HP. LP arrivals come with rate $\boldsymbol{\lambda_L}$ and are served at rate $\boldsymbol{\mu_L}$. HP arrivals come at rate $\boldsymbol{\lambda_H}$ and are served at rate $\boldsymbol{\mu_H}$. The queue has the following special rules –

(i) A HP arrival forces the queue to throw out the LP jobs that are currently in the system (waiting in queue or in service). This will empty all LP jobs in the system and will even prematurely terminate an LP job if it is currently in service – these discarded or prematurely terminated LP jobs do not receive any further service

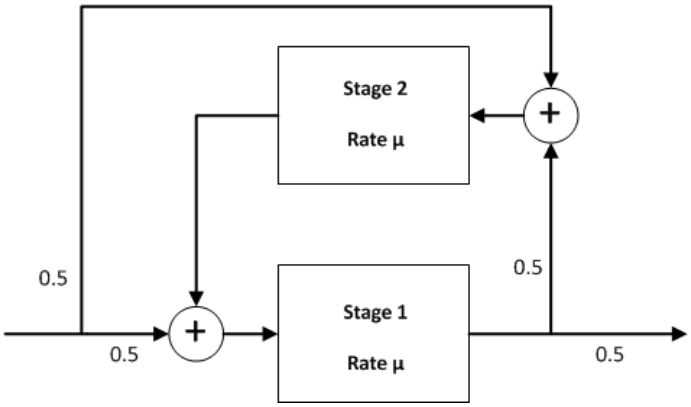
(ii) A LP job, for which service is started, completes its service only if no HP jobs arrive during its service time

(ii) LP arrivals are allowed to enter the system only when there are no HP jobs in the system and the system is not full (i.e. has less than two LP jobs). LP jobs which are not allowed to enter the system leave without service

(iii) At any time, **only one HP job** can be in the system (i.e. in service). Any other HP arrivals coming during a HP service time leaves without service

(a) Draw a State Transition diagram with an appropriate definition of the system state. **[4]**
(b) What is the probability that there are no HP jobs in the system? **[2]**
(c) What is the probability that the system is empty? **[5]**

2. Consider a service facility whose service can be modelled by stages as shown. Each service stage provides an exponentially distributed service time with rates as shown in the figure. This service facility is the server for an M/-/1 queue which gives service to arrivals coming at rate λ.



(a) What would be the distribution of the overall service time for a job? **[5]**
Note: *Give any of pdf, cdf or LT (of pdf)*
(b) What is the condition for the queue to be stable? **[3]**
(c) Draw the State Transition Diagram for the system **[5]**

3. In an M/G/1 queue, the server goes for a **single vacation with probability *p*** when the system goes idle following a busy period. Assuming standard notation, do the following.

(a) What is the probability **P{server on vacation}**? **[5]**
(b) Using the *Residual Life Approach*, find the **Mean Number in the Queue** waiting for service. **[5]**
(c) Using the *Imbedded Markov Chain Approach*, obtain the **distribution** for the number in the system. **[6]**