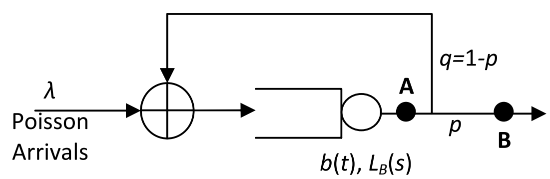
EC 633, Queueing Systems
Home Assignment No. 5

Date/Place of Submission: Lecture of 30-SEP-2009 (*In view of our past experience, your TAs will collect your home assignments from you as you enter the lecture.*)

1. Complete the derivation for the **M/G/1 queue with only one vacation interval per idle** as discussed in class. Do it using both the Imbedded Markov Chain approach and the Residual Life approach and verify that the two give the same average performance measures.

2. Complete the derivation for the **M/G/1 queue with exceptional first service** as discussed in class. Do it using both the Imbedded Markov Chain approach and the Residual Life approach and verify that the two give the same average performance measures.

3. Consider the queueing system shown below where the queue has infinite buffers and provides service with service time distribution $b(t)$, $L_B(s)$. On service completion at the queue, the customer randomly decides to leave the system (with probability p) or rejoin the queue for another round of service (with probability q=1-p). Assume that the rejoining customer gets served ahead of other customers, if any, who may be waiting in the queue for service. The arrivals to the overall system come from a Poisson process with average arrival rate λ.



   (a) Consider the whole system as a single M/G/1 queue. What would be its effective service time distribution (and mean)? Analyse this system using an *imbedded Markov chain* approach and obtain the state distribution at the departure instants as seen at point **B** indicated in the figure (i.e. on departure from the overall system).

   (b) Consider the system at point **A** in the figure. Analyse the *imbedded Markov chain* at the departure instants at this point and obtain the corresponding state distribution as seen by the customer departing from the internal queue.

4. Consider arrivals coming in a random time interval (pdf $b(t)$, cdf $B(t)$ and L.T. $L_B(s)$) from a Poisson process with rate λ. Define A_k as the probability of there being k or more arrivals in such a time interval. Show analytically that –

$$A_k = \int_0^\infty \frac{(\lambda x)^{k-1}}{(k-1)!} e^{-\lambda x} [1 - B(x)] \lambda dx \qquad \text{for } k = 1,2,.......\infty$$

Hint: You may find the following useful $\int e^{-\lambda x} b(x) dx = e^{-\lambda x} B(x) + \lambda \int e^{-\lambda x} B(x) dx$