

EC 633, Queueing Systems Home Assignment No. 4

Date/Place of Submission: Lecture of 09-SEP-2009

1. For the $M^{[X]}/M/1$ queue discussed in class (with batch arrival rate λ , service rate of individual jobs as μ , Generating Function of the batch size as $\beta(z)$) show that the mean number of jobs in the system will be given by

$$\bar{N} = \frac{\rho(\bar{\beta} + \bar{\beta}^2)}{2\beta\left(1 - \frac{\lambda\bar{\beta}}{\mu}\right)}$$

If the queue operates in a FCFS fashion, what will be the mean delay before service to a batch starts? (You need to use the M/G/1 results to answer this.)

Note: It may be interesting to modify the queue discussed in class to include the possibility that a batch may have no customers with probability β_0 (i.e. an empty batch). How does the analysis change if we use this model?

2. Consider a $M^{[X]}/M/1/4$ queue with batch arrival rate λ and service rate of individual jobs as μ . The Generating Function of the batch sizes is given to be $(\beta_0 + \beta_1z + \beta_2z^2)$. Analyse this system for the cases where (i) PBAS and (ii) WBAS strategies are being used. For each case, obtain the following.
 - (a) State probabilities of the system
 - (b) The mean number in the system
 - (c) The probability that a batch is refused entry in the WBAS case and the probability that a job is refused entry in the PBAS case.
3. Consider a $M^{[X]}/-/1/3$ queue where the batch arrival rate is λ and the generating function of the batch sizes is given by $(0.25+0.25z+0.5z^2)$. We assume that the queue follows the WBAS strategy.

(a) The L.T. of the service time distribution is given as -

$$L_B(s) = \frac{0.5\mu}{s + \mu} + \frac{0.5\mu^2}{(s + \mu)^2}$$

For this, draw the state transition diagram of the queue with an appropriately defined system state.

(b) For a service time distribution with L.T. $\mu/(s+\mu)$, do the following.

- i. Draw the state transition diagram.
- ii. Find the state probabilities of the queue.
- iii. What is the probability that a batch is refused entry into the queue?