EC 633, Queueing Systems
Home Assignment No. 3

Date/Place of Submission: *Not to be submitted*

1. Consider the M/- /1/3 queue, which is limited to having a maximum of 3 users in the system. Assume that the arrivals come from a Poisson process of average rate λ and that the L.T. of the service time distribution is

$$L_B(s) = \frac{2\mu^2}{(s+\mu)(s+2\mu)}.$$

(a) Draw the *State Transition Diagram* of the system and write its balance equations.
(b) Use (a) to find the probabilities of the individual states in the state transition diagram.
(c) Use (b) to find the overall state probabilities p_0, p_1, p_2 and p_3.
(d) What is the probability that an arrival will have to leave without service?

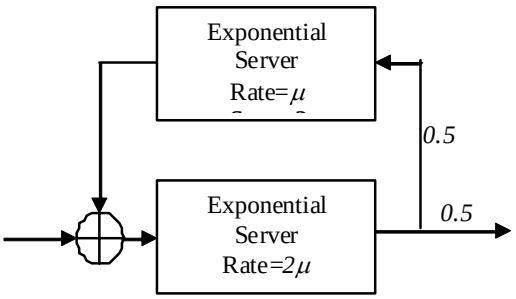
2. A M/-/1/2 queue has a service time distribution with L.T. given by

$$L_B(s) = \frac{0.5\mu}{s+\mu} + \frac{0.5\mu^2}{(s+\mu)^2}$$

The average arrival rate is λ. Note that the queue is limited to a maximum state of 2. Use the method of stages to solve this queue and obtain the following.

(a) State Transition Diagram (with a proper definition of system states)
(b) Obtain the state probability distribution
(c) What will be the average departure rate from this queue?

3. Consider a M/-/1/2 queue where the service facility is modelled as shown below. After finishing service at Stage 1, the job either exits the system with probability 0.5 or moves to Stage 2 with probability 0.5, as shown. Note that a job entering the service facility always gets served at *Stage 1* first. Assume that the arrival rate to the queue is λ.



(**a)** Draw an appropriate state transition diagram for the system.
(b) Write the balance equations and solve these for the state probabilities using $\rho=\lambda/\mu$
(c) What is the mean queueing delay seen by an arrival entering the system?
(d) What is the effective service time distribution of the server?