EC 633, Queueing Systems
Home Assignment No. 2

Date/Place of Submission: Lecture of 24-AUG-2009

1. **Exceptional First Service:** *Some systems require a set-up to be followed whenever the system is empty. This implies that the service rate of the first customer being served after the system becomes idle is different from the "typical" service rate.*

   Consider a M/M/1 queue where the mean service time is $\Delta+\mu^{-1}$ for the first customer being served after the system becomes empty whereas it is μ^{-1} for all other cases. If the arrival rate is λ, obtain the state probability distribution under equilibrium conditions.

2. Consider a M/M/1 queue where service is provided with rate μ and new arrivals come at rate λ. However, if an arrival finds the queue to be in state j then it decides not to join the queue with probability $1\text{-}e^{-\alpha j}$, i.e. leaves the queue without service.

   Find the state probabilities of this system under equilibrium conditions and the average rate of customers who leave the queue without service. What would be the system behaviour as $\alpha\rightarrow\infty$ and what would such a system physically correspond to?

3. Consider a M/M/2 queue with two servers A and B with respective service rates μ_A and μ_B where the service times are exponentially distributed. If an arrival finds the system empty then it gets served by the server which has been idle for the maximum time since when it was last busy. In all other situations, the system behaves like a normal M/M/2 queue, i.e. customers are served in a FCFS fashion by whichever server becomes available.

   Draw a state transition diagram for the system with an appropriate definition of the system states. Solve this for the system state probabilities and use them to get the probabilities of finding i users in the system for i=0,1,2,........

4. **Server Vacations:** The clerk sitting at the railway reservation counter in Guwahati station serves customers with an exponentially distributed service time μ^{-1} . Arrivals to his counter come from a Poisson process with rate λ. On Mondays, Tuesdays and Wednesdays, he works like a normal M/M/1 system. On Thursdays and Fridays, he goes for a tea break whenever the queue becomes empty where the tea-break is exponentially distributed with mean β^{-1}. When he returns from the tea-break, he resumes service normally if there are people waiting for service in the queue; otherwise, he goes for another tea-break and keeps doing this until he comes back from a tea-break and finds customers waiting for service. (*He does not work on weekends! Needs a break like all of us*!) Miss Mamata Banerjee has approached you for an official assessment on whether she should pay the clerk more for his work on Monday-Wednesday. What would you recommend and why? Justify your answer quantitatively.