EC 633, Queueing Systems
Home Assignment No. 1

Date/Place of Submission: Lecture of 17-AUG-2009

1. A radioactive sample emits α-particles at the rate λ particles per second where the time interval between the emissions of successive particles is given to be exponentially distributed with mean λ^{-1}. A counter (initialized to zero) is used to count the number of particles emitted. Show that the probability of the counter detecting k particles in time T is given by the Poisson distribution, i.e. $P_k(T) = e^{-\lambda T}\frac{(\lambda T)^k}{k!}$ for k=0,1,2,.....
   (Note that this shows that when the inter-arrival times have an exponential distribution, the corresponding arrival process will be Poisson.)

2. A machine shop has two machines MC1 and MC2. The time for MCi to break down is an exponentially distributed random variable with mean μ_i^{-1} i=1, 2. Once a machine breaks down, we start repairing it immediately. For both MC1 and MC2, the time to repair it is an exponentially distributed random variable with mean μ^{-1}.

   (a) Consider a time instant when both MC1 and MC2 are working. What will be the probability that MC1 will break down first?
   (b) Consider a time instant when MC1 has broken down but MC2 is working. What will be the probability that MC2 will break down before MC1 is repaired?
   (c) Compute probabilities of the following under equilibrium conditions –

      i. both machines are working
      ii. neither machine is working
      iii. MC1 is working,
      iv. MC2 is working
      v. MC1 is working but MC2 is under repair
      vi. MC1 is under repair but MC2 is working

3. Prof. Calculus makes it a point to start his office hours at exactly 8:00 am on Wednesday mornings but can only handle one student at a time in his office. Each student stays in his office for a random duration X. Students arriving when there is already one student in his office, wait outside. Students arrive following a Poisson process with average arrival rate λ.
   (a) If $X=c$ (constant), what is the probability that the *second arriving* student will not have to wait? In this case, what will be the mean waiting time of this student (i.e. the one who arrives second)?
   (b) Repeat (a) when X is exponentially distributed with mean μ^{-1}.

4. A service facility has K servers where the servers are identical but work independently from each other. The facility does not have any additional place for customers to wait in case they arrive and find all servers busy. A server engaged in service provides a service time which is exponentially distributed with mean μ^{-1}. Customers arrive to this service facility following a Poisson arrival process (i.e. exponentially distributed inter-arrival times) with rate λ. Assume that at time t=0, the manager of the service facility inspects the system and happily notes that all the servers are engaged.
   (a) What is the probability that the next customer arriving to this service facility finds all servers busy and leaves without service?
   (b) What is the probability that the next customer arriving to this service facility finds *at least* two servers free?
   (c) What is the mean number of customers who will get turned away before any of the servers become free?