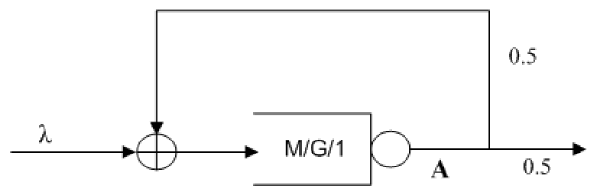
EC 633, Queueing Systems
Final Examination

Maximum Marks = 100 *Attempt all questions,* ***Read questions carefully***

1. For the M/G/1 queue shown in the figure, assume that the service time of the queue itself has mean $\overline{X}$ with pdf, cdf and the L.T of the pdf as $b(x), B(x)$ and $L_B(s)$, respectively. Use $\rho = \lambda \overline{X}$ for notational convenience.



(a) What is the probability that a departing customer at point A will see the system empty? **[5]**
(b) Derive the generating function of the number in the system as seen by a departing customer at point **A**. **[5]**

2. The mean arrival rate to an M/G/1/1 queue is λ and the mean service time is $\overline{X}$. What will be the probability that an arrival to this queue will be blocked? **[5]**

3. In an M/G/1 queue, once the queue becomes empty, service starts again only after *K* new jobs arrive. **In addition**, the service is such that the first service time in the busy period requires an extra time Δ (fixed). Let *λ* be the average arrival rate of jobs to the system and let $\overline{X}$ be the normal mean service time. (The first job in the busy period will have a mean service time of $\overline{X}$ + Δ.). What will be the probability of finding the server busy in this queue?
Use $\rho = \lambda \overline{X}$ for notational convenience. **[10]**

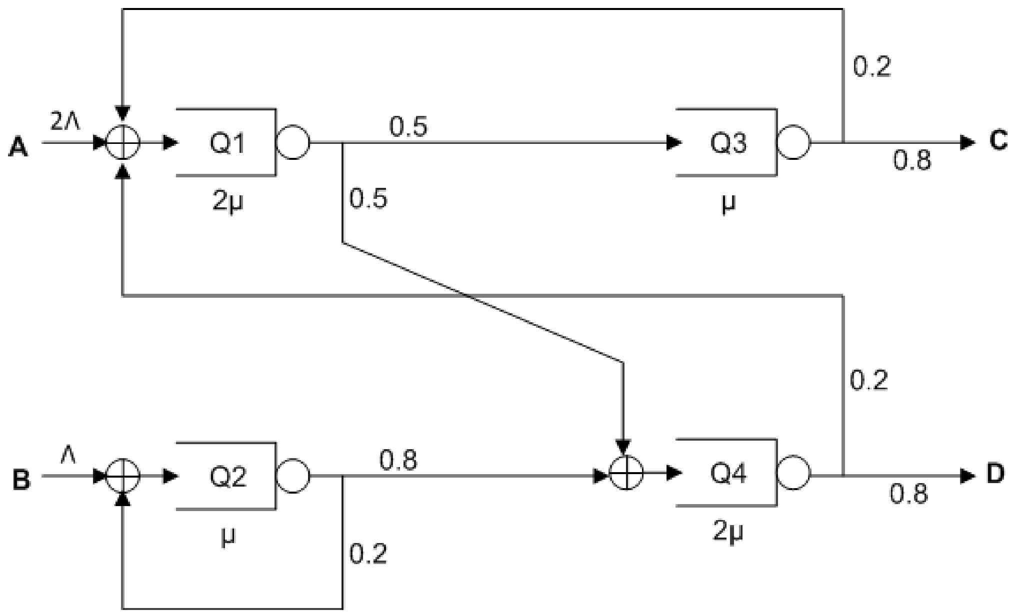
4. An M/D/1/2 queue has arrivals coming at rate *λ* where jobs which are blocked leave the system without service. The service time is of fixed duration *T*.

(a) Obtain the state probabilities as seen by a customer departing after service (i.e. the state probability as seen by that customer looking back into the system). **[5]**
(b) Derive the blocking probability P_B, i.e. the probability that an arriving customer will find the queue full and will leave without service. **[10]**

5. Consider a Geo/G/1/2 queue where a packet arrives in a slot with probability *λ* and the probability of no packets arriving in a slot is (1-*λ*). Service time for each job takes a random number of slots where b_k is the probability that the service time will be of *k* slots (*k*=1,2,3........) with generating function $B(z) = \sum_{k=1}^{\infty} b_k z^k$.

(a) Using the **Late Arrival Model**, derive the probabilities for the number in the queue at the job departure instants (i.e. just after the departure). **[10]**
(b) Using the **Early Arrival Model**, derive the probabilities for the number in the queue at the job departure instants (i.e. just after the departure). **[10]**

6. Consider the open queueing network of single server queues with external arrivals, routing probabilities and service rates of the queues as shown. Assume that the external arrivals come from a Poisson process and that the service time distributions are exponential in nature.

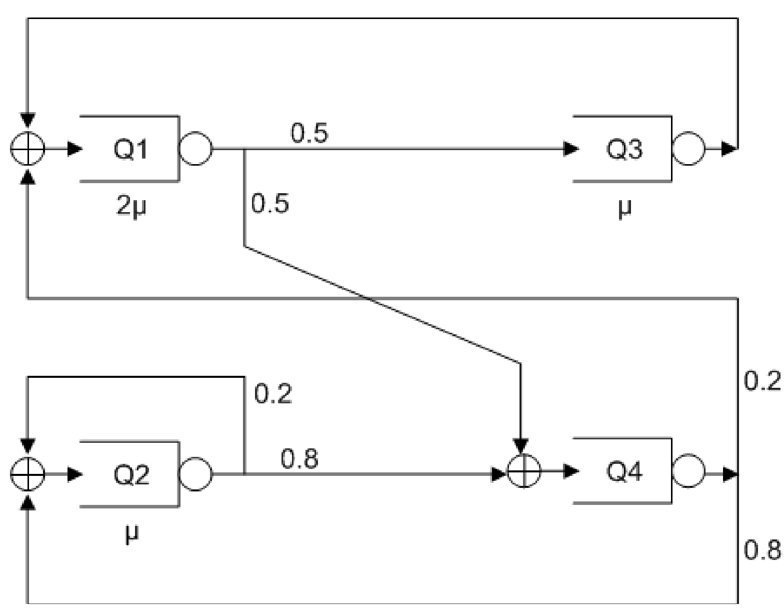


(a) What will be the condition for the network to be stable? **[5]**

(b) What will be the transit delay through the system for any job arrival when **Λ=0.5 and µ=1**? **[5]**

(c) What will be the transit delay through the system for a job entering the network at **A** when **Λ=0.5 and µ=1**? **[10]**

7. Consider the closed queueing network of single server queues (with exponentially distributed service times) with service rates as shown. **Assume µ=1**



We wish to use Norton's Theorem for analyzing the performance of different Q4 in this network when there are **M=4** jobs circulating in the system. This requires computing the Flow Equivalent Server of the network for Q1, Q2 and Q3 where the designated sub-network is the queue Q4. Compute the service rates $\mu(j)$ for this FES for j=1,2,3,4 **[20]**