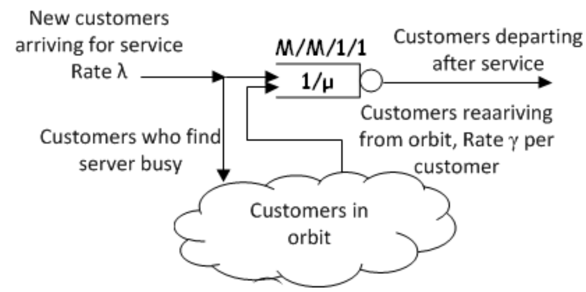
EE633 2014-2015F
Final Examination

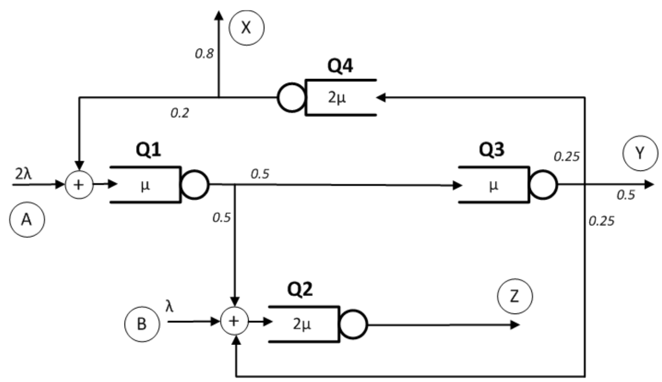
Time: 3 hours *Marks 50*

1. For the retrial queue shown, the system state is (j, n) *where* j is the number in the M/M/1/1 queue and n is the number in orbit. If there are n customers in orbit then the rate of customer arrivals to the M/M/1/1 queue will be $n\gamma$. For this queue, derive the following. *Use* $\rho=\lambda/\mu$**.** (*Proper steps must be shown. No marks will be given if intermediate steps are omitted.*)

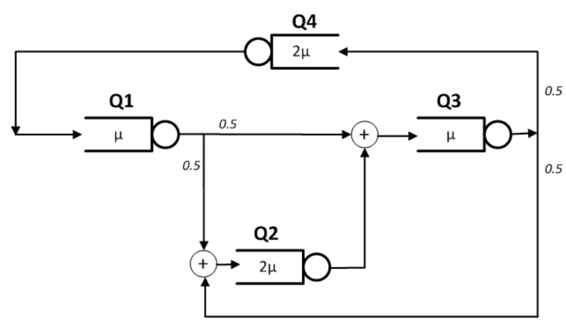


(a) The generating function $P(z)$ of the number of customers in the orbit **[8]**
(b) The mean number of jobs in orbit **[5]**
(c) The mean number of jobs in the queue **[2]**

2. Consider the open network of M/M/1 queues shown, where we use $\rho=\lambda/\mu$ for notation.
(a) What is the limiting value of ρ for the system to be stable? **[2]**
Do the following for λ=0.4 and μ=1.
(b) Find the mean number in each queue. **[4]**
(c) Mean Transit Delay through the system for a job entering at A **[4]**



3. Assume that there are M=4 jobs circulating in the closed queueing network shown. We want to apply *Norton's Theorem* to this network, where we consider **Q4 to be the target queue**. Assume μ**=1.0**
(a) Find the **Flow Equivalent Server (FES)** for M=4 for the rest of the network (i.e. Q1, Q2 and Q3). **[10]**
(b) Analyse the equivalent network consisting of the FES and Q4 to obtain probabilities of finding k jobs in Q4, k=0, 1, 2, 3, 4. **[5]**



4. Consider a **M/G/1 queue with Vacations** where arrivals come at rate λ. The usual service time is X (mean $\overline{X}$, second moment $\overline{X^2}$, pdf $b(t)$, LT of pdf $L_B(s)$) and each vacation interval is V (mean $\overline{V}$, second moment $\overline{V^2}$, pdf $f_V(t)$ and LT of pdf $L_V(s)$). In addition, the **first service time** when service is resumed (after the server goes idle) has an additional **fixed** component Δ. In this system, the server goes on vacation whenever the system becomes idle. It resumes service **only when it comes back from a vacation and finds at least one job waiting in the queue**; otherwise it goes for another vacation. For this queue, find the following
(a) Mean Length $\overline{BP}$ of the Busy Period **[3]**
(b) Mean Length $\overline{T_{cycle}}$ of Busy-Idle Cycle **[3]**
(c) Mean Number $\overline{N_{BP}}$ served in a busy period **[3]**
(d) Mean Effective Service Time (taking the exceptional first service into account) **[1]**