EE633 Queueing Systems (2016-17F)
End Semester Examination

Maximum Marks: 50 Time: 180 minutes

The total marks in this question paper is 60 but the marks credited will be saturated at 50.

1. Consider a **2-priority, pre-emptive resume M/G/1** queue where Class 2 has higher priority than Class 1. The queue restricts the **maximum number of Class 2 jobs in the system to two (2)**, i.e. one in service and at most one more in the buffer. However, it can buffer an **infinite number of Class 1 customers**.

Customers of Class *j* arrive from a Poisson process at rate λ_j and require service given by the random variable X_j $j=1,2$. The service times have mean $\overline{X_j}$, second moment $\overline{X_j^2}$, pdf $b_j(x)$ and L.T. of the pdf as $L_{Bj}(s)$ for $j=1,2$

(a) What are the equilibrium probabilities of finding *n*, *n*=0, 1, 2, Class 2 customers in the system?
(*Hint: Note that Class 2 customers have preemptive resume priority over the Class 1 customers but that at most two Class 2 customers can be in the system!*) **[4]**

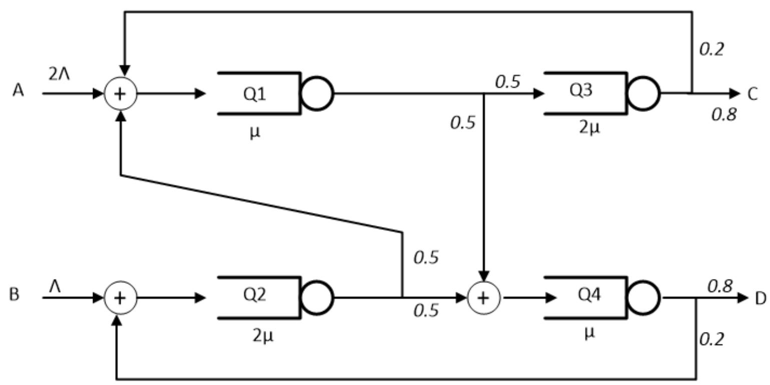
(b) What would be the condition on $\rho_1 = \lambda_1 \overline{X_1}$ for the system to be stable for Class 1 customers? **[1]**

(c) Find the mean and distribution (pdf or L.T. of pdf) of the busy period of the queue for Class 2 customers **[1+4]**

(d) A Class 1 customer starts service at time *t*=0 and finally finishes its service at time *T*. Find the mean and distribution (pdf or L.T. of pdf) of the random variable *T*. **[2+4]**
(*Hint: The random variable T would be the effective service time of a Class 1 customer which would consist of its actual service times* X_1 *and the periods for when the Class 1 service is preempted. Note also that whenever there is a Class 2 arrival while the Class 1 customer is being served, a Class 2 busy period starts whose length would be the length of that particular preemption period.*)

(e) What would be the **equilibrium state distribution of Class 1 customers** in this system? **[4]**
(*Hint:* $P(z) = \dfrac{(1-\rho)(1-z)A(z)}{A(z)-z}$ *for the standard M/G/1 queue with proper definitions of* ρ *and* $A(z)$.)

2. Consider the open network of **M/M/1** queues shown in the figure. Jobs enter the system from **A** or **B** with rates 2Λ and Λ, respectively, as shown. The service rates for **Q1, Q2, Q3** and **Q4** are μ, 2μ, 2μ and μ and the routing probabilities are as shown in the figure.



(a) When will the system be stable? **[2]**

(b) Assuming Λ = 0.3 and μ= 1 find the following

- **(i)** What is the mean transit time through the system? **[2]**
- **(ii)** What is the mean transit time through the system for a job arriving at **B**? What is the mean transit time for a job arriving at **A**? **[3+3]**
- **(iii)** What is the mean transit time through the system for a job arriving at **B** and leaving from **C**? What is the mean transit time for a job arriving at **B** and leaving from **D**? **[3+3]**

(c) Draw the **reversed queueing network for Λ=0.3 and μ=1**, where the jobs arrive from **C** and **D** and leave from **A** and **B**. In this diagram, write the respective flow rates entering from **C** and **D** and leaving from **A** and **B**, the flows through each queue and the routing probabilities wherever the flows are split. **[4]**

3. Consider a Geo/G/1 queue where the probability of customer arrival in a slot is given to be λ. The probability that a job will require k slots for service is $b(k)$ with $b^{(j)}$ j=1, 2, …. as the j^{th} moment of the service time (in slots) and $B(z)$ as the corresponding generating function.

(a) Use the **Late Arrival Model** to derive the Generating Function $P(z)$ of the number in system at the customer departure instants and use this to find the corresponding mean number in the system. **[6+6]**

(**Note:** *All steps must be shown. DO NOT assume any prior result.*)

(b) If we had considered the *Early Arrival Model*, we could have derived the corresponding Generating Function $\tilde{P}(z)$ for that model. Note that even though $P(z) \neq \tilde{P}(z)$, both models should give the same results for the distribution of the number of slots that an arrival spends in the system, where the time spent in system is suitably defined. **Use this** to get an expression for $\tilde{P}(z)$ in terms of $P(z)$, where the queue is assumed to be FCFS in nature. **[8]**

(**Note:** No marks will be given if you derive $\tilde{P}(z)$ directly as was done for getting $P(z)$ in (a))