EE633 Queueing Systems (2015-16F)
End Semester Examination

Maximum Marks: 100 Time: 180 minutes

Please spend 5 minutes to read and understand the questions!

1. Your local branch of the Service-First Bank (SFB) has one teller window. Customers come for teller service at rate λ from a Poisson process and the bank provides an infinite number of chairs for people to wait. Customer service times are exponentially distributed with mean 1/μ. Unfortunately, the teller is somewhat temperamental and decides to take a break with probability $\gamma\Delta t$ in time interval Δt (this happens while it is actually serving) with $\Delta t \to 0$. When that happens, the manager himself steps in to continue service to the customer who was currently being served, where the manager serves in the same way, i.e. exponential with rate μ, as the teller. The teller resumes normal operation again after that service (i.e. provided by the manager) completes but can of course take another break with probability $\gamma\Delta t$ in the same way once again while serving another customer. When the system becomes empty, i.e. state is zero with probability p_0, then the teller stays waiting to start service to the next customer entering the bank.

To model this system, represent the system state as {*n*} *n*=1,2,…. when the teller is serving and {*n**} *n**=1,2, ….. when the manager is working. The corresponding state probabilities are p_n and p_n^*, respectively. Their respective generating functions are defined as $P(z) = \sum_{n=1}^{\infty} p_n z^n$ and $P^*(z) = \sum_{n=1}^{\infty} p_n^* z^n$.

[**Note:** $P(z)$ does not include the p_o term. This will affect how you write the Normalization Condition!]

For a system in equilibrium, do the following. (**For notational convenience, use** $\rho = \frac{\lambda}{\mu}$ **and** $\alpha = \frac{\gamma}{\mu}$.)

(a) Draw the State Transition Diagram of the system **[5]**

(b) Write the balance equations for the system. **[5]**

(c) Use the balance equations to find expressions for the generating functions $P(z)$ and $P^*(z)$ in terms of p_0, ρ and α **[5]**

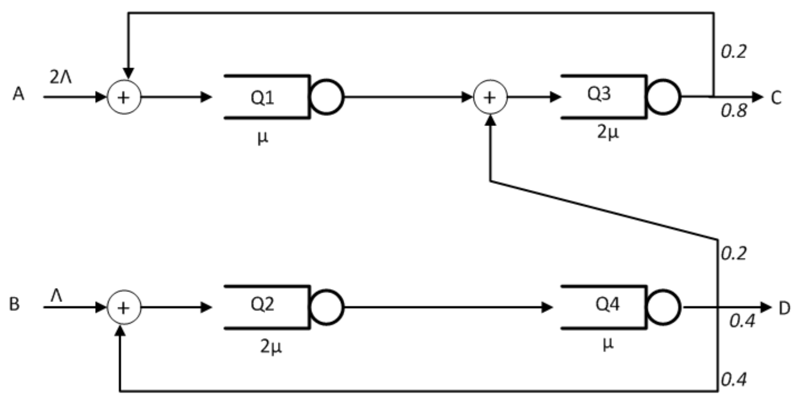
(d) Apply the normalization condition to obtain p_0, the probability of the system being empty. **[5]**

(e) From (c) and (d), derive the *probability that there are n customers in the system* **[5]**

(f) Find the *probability that the manager is working* and the *probability that the teller is working*. **[5]**

(g) How will you physically interpret the quantities $\overline{n} = \sum_{n=1}^{\infty} np_n$, $\overline{n^*} = \sum_{n=1}^{\infty} np_n^*$ and ($\overline{n} + \overline{n^*}$) **[5]**

2. Consider the open network of **M/M/1** queues shown in the figure. Jobs enter the system from **A** or **B** with rates 2Λ and Λ, respectively, as shown. The service rates for **Q1, Q2, Q3** and **Q4** are μ, 2μ, 2μ and μ and the routing probabilities are as shown in the figure.



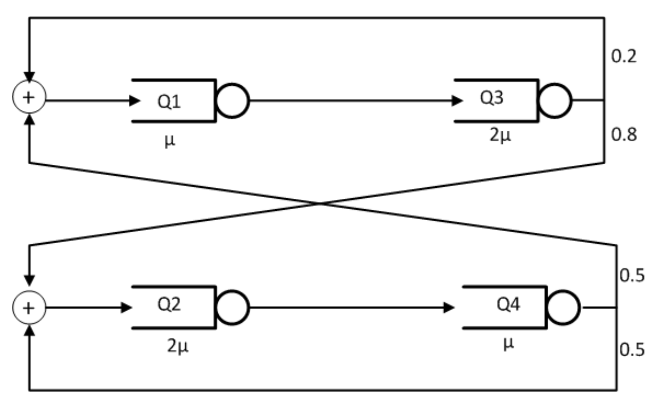
(a) When will the system be stable? **[5]**

(b) Assuming Λ = 0.3 and μ= 1 find the following

- **(i)** What is the mean transit time through the system? **[5]**
- **(ii)** What is the mean transit time through the system for a job arriving at **B**? What is the mean transit time for a job arriving at **A**? **[8+2]**
- **(iii)** What is the mean transit time through the system for a job arriving at **B** and leaving from **C**? What is the mean transit time for a job arriving at **B** and leaving from **D**? **[8+2]**

(c) Draw the **reversed queueing network for Λ=0.3 and μ=1**, where the jobs arrive from **C** and **D** and leave from **A** and **B**. In this diagram, write the respective flow rates entering from **C** and **D** and leaving from **A** and **B**, the flows through each queue and the routing probabilities wherever the flows are split. **[5]**

3. Assume that there are M=4 jobs circulating in the closed queueing network shown. The queues are single server queues with exponentially distributed service times with service rates as shown for each queue. We want to apply *Norton's Theorem* to this network, where we consider **Q4 to be the target queue**. Assume $\boldsymbol{\mu}$**=1**



(a) Find the **Flow Equivalent Server (FES)** for M=4 for the rest of the network (i.e. Q1, Q2 and Q3). **[20]**

(b) Analyze the equivalent network consisting of the FES and Q4 to obtain the probabilities of finding k jobs in Q4 for k=0, 1, 2, 3, 4 **[10]**