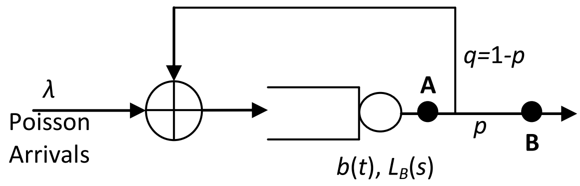
EE 633, Queueing Systems
Final Examination (2012-2013S)

Max Marks 50 **Attempt all questions**

1. Consider the queueing system shown where the queue has infinite buffers and provides service with service time distribution $b(t)$, $L_B(s)$. On service completion at the queue, the customer randomly decides to leave the system (with probability p) or rejoin the queue for another round of service (with probability q=1-p). Assume that the rejoining customer gets served ahead of other customers, if any, who may be waiting in the queue for service. The arrivals to the overall system come from a Poisson process with average arrival rate λ.



(a) Consider the whole system as a single M/G/1 queue. What would be its effective service time distribution (and mean)? Analyse this system using an *imbedded Markov chain* approach and obtain the state distribution at the departure instants as seen at point **B** indicated in the figure (i.e. on departure from the overall system). **[5]**

(b) Consider the system at point **A** in the figure. Analyse the *imbedded Markov chain* at the departure instants at this point and obtain the corresponding state distribution as seen by the customer departing from the internal queue. **[5]**

2. Consider a M/M/2 queue with two servers A and B with respective service rates μ_A and μ_B where the service times are exponentially distributed. If an arrival finds the system empty then it gets served by the server which has been idle for the maximum time since when it was last busy. In all other situations, the system behaves like a normal M/M/2 queue, i.e. customers are served in a FCFS fashion by whichever server becomes available. Assume arrivals come at rate λ.

(a) Draw a state transition diagram for the system with an appropriate definition of the system states. **[3]**
(b) Obtain the system state probabilities as per your definition in (a) **[5]**
(c) Obtain the probabilities of finding k jobs in the system for k = 0, 1, 2,........ **[2]**

3. **(a)** The mean arrival rate to an M/G/1/1 queue is λ and the mean service time is $\overline{X}$. What will be the probability that an arrival to this queue will be blocked? **[2]**

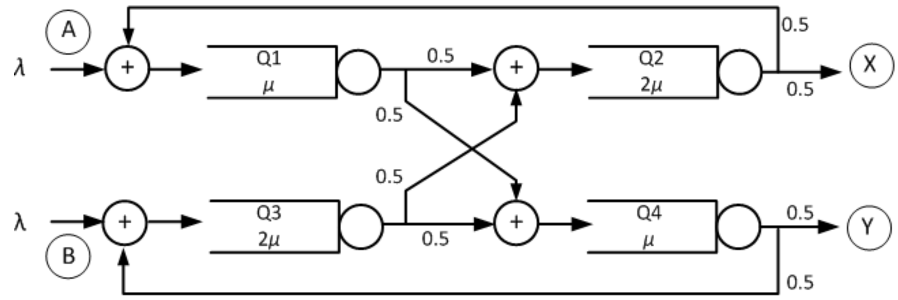
(b) In an M/G/1 queue, once the queue becomes empty, service starts again only after K new jobs arrive. **In addition**, the service is such that the first service time in the busy period requires an extra time Δ (fixed). Let λ be the average arrival rate of jobs to the system and let

$\overline{X}$ be the normal mean service time. (The first job in the busy period will have a mean service time of $\overline{X} + \Delta$.). Use $\rho = \lambda \overline{X}$ for notational convenience.

(i) What will be the probability of finding the server busy? **[5]**

(ii) What will be the probability of finding the system empty? **[3]**

4. For this network of M/M/- type queues, find the following.

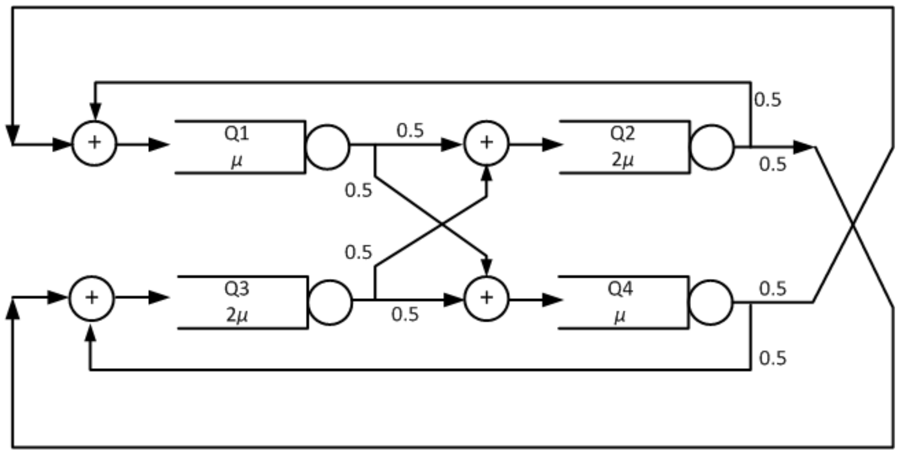


(a) When will the system be stable? **[2]**

(b) What is the mean transit time through the system for **λ=0.1 and μ=1**? **[3]**

(c) For **λ=0.1 and μ=1**, what will be the transit time through the system for a job entering the network at **A**? **[5]**

5. Assume that there are M=4 jobs circulating in the closed queueing network shown. We want to apply *Norton's Theorem* to this network, consider *Q4* to be the target queue. Assume **μ=1.0**



(a) Find the **Flow Equivalent Server (FES)** for M=4 for the rest of the network (i.e. Q1, Q2 and Q3). **[5]**

(b) Analyze the equivalent network consisting of the FES and Q4 to obtain probabilities of finding k jobs in Q4, k=0, 1, 2, 3, 4. **[5]**