EE 633, Queueing Systems (2017-18F)
End Semester Exam

*Maximum Marks: 100 (**will be scaled to 50**)* *Time: 3 hours*

1. Consider a M/M/2 queue with two servers A and B where the arrivals come from a Poisson process at rate λ. Server A works at rate μ_A and server B works at rate μ_B. Once the system becomes empty, the next arrival is served by the server who was working last with probability α and by the other server with probability (1-α).

Define the system state as follows –

N=2, 3,….. System States (number in system) as usually defined for 2 or more customers in the system

1A One customer in the system being served by Server A

1B One customer in the system being served by Server B

0A System empty, Server A was the one who was working last

0B System empty, Server B was the one who was working last

(**a**) Draw the State Transition Diagram of the system at equilibrium. **[5]**

(b) Write the balance equations that will be needed to solve for the probabilities of the system state (as defined above). **[5]**

(c) Solve the balance equations to obtain the probabilities of each system state (as defined above) **[25]**

(**Important: Marks will be given only for properly and adequately simplified expressions** as the results do show something interesting!! Can you see what? **<u>10 BONUS MARKS</u> for telling me what that is!**)

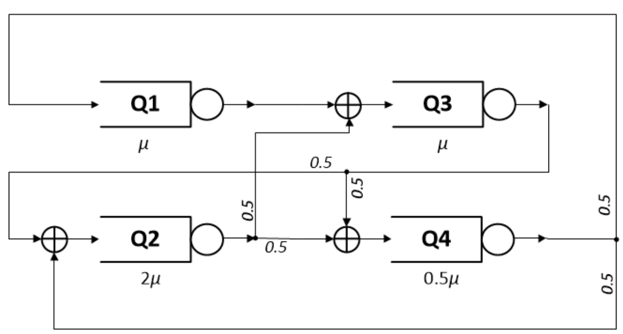
2. Consider a FCFS M[X]/G/1 queue where the arrivals come in batches where there are either one or two jobs in the batch with equally likely probabilities, i.e. P{Batch Size = 1} = P{ Batch Size = 2} = 0.5. The **first** job of the batch has a random service time with its n^{th} moment given to be $\alpha(n)$ and L.T. of its pdf as $L_\alpha(s)$. The **second** job of the batch (if any) has a random service time with its n^{th} moment given to be $\beta(n)$ and L.T. of its pdf as $L_\beta(s)$. *The two random variables are independent of each other.*

(a) What will be the mean queueing delay for an arbitrary job (first or second in a batch) and the L.T. $L_{Wq}(s)$ of its pdf? **[10+10]**

(b) What will be the mean queueing delay observed by the second job in a batch with two jobs. **[5]**

(*Note: Standard results for the basic M/G/1 queue (using standard notation), may be stated properly and used directly, i.e. there is no need to derive these if you can state them correctly.*)

3. (a) For **any** closed network of queues with M jobs, where n_i is the number of jobs in the i^{th} queue, show that $\sum_{n=1}^{M} nP\{n_i = n\} = \sum_{n=1}^{M} P\{n_i \geq n\}$ **[5]**



(b) For M=4 jobs circulating in the closed queueing network shown, we want to apply *Norton's Theorem* to this network, where we consider **Q4 to be the target queue**. Assume $\boldsymbol{\mu}$=1

(i) For M=4, find the **Flow Equivalent Server (FES)** for the rest of the network (i.e. Q1, Q2 and Q3). **[20]**

(ii) Analyze the equivalent network consisting of the FES and Q4 to obtain **[10+5]**

(A) The probability of finding k jobs in Q4, k = 0, 1, 2, 3, 4

(B) The Mean Transit Time through Q4

(**<u>10 BONUS MARKS</u> if you verify your answer for (B) above through MVA or Convolution)**