EEE33 Queueing Systems (2011-12)
Final Examination

Maximum Marks: 50 **Time: 3 hours**

1. Consider a 2-priority, pre-emptive resume M/G/1 queue where Class 2 has higher priority than Class 1. The queue imposes the restriction that there can be **at most two (2) Class 2 jobs** in the system at any given instant, i.e. one in service and at most one more in the buffer. However, Class 1 jobs have an infinite buffer.

Use standard notation, i.e. $\{\lambda_i, \overline{X}_i, b_i(x), B_i(x), L_{Bi}(s)\}$ for class *i*, *i*=1, 2

(a) What are the equilibrium probabilities of finding *j*, *j*=0,1,2, class 2 customers in the system? **[4]**

(b) Find the mean and distribution (L.T. of pdf) of the busy period of the queue for class 2 customers. **[1+2]**

(c) A class 1 customer starts service at time *t*=0 and finishes its service at time *T*. Find the mean and distribution (L.T. of pdf) of *T*. **[1+2]**

2. Consider a 2-priority, pre-emptive priority M/M/1 queue where at any time **at most one high priority job** can be in the system. However, the queue allows infinite buffering for the lower priority class. High priority arrivals coming when another high priority job is in the system are blocked and are forced to leave without service. Let λ_H and λ_L be the average arrival rates of high and low priority jobs to the system and let μ_H and μ_L be their mean respective service rates. Define the system state as (n_H, n_L) with state 0 to be the state when the system is empty.

(a) Draw the state transition diagram for the system. **[2]**

(b) What is the probability that the server is idle? **[3]**

(c) What is the blocking probability for high priority jobs? **[3]**

(d) What is the probability of finding **one job** in the system? *[Note that for this part, simplification of the final expression is not required.]* **[2]**

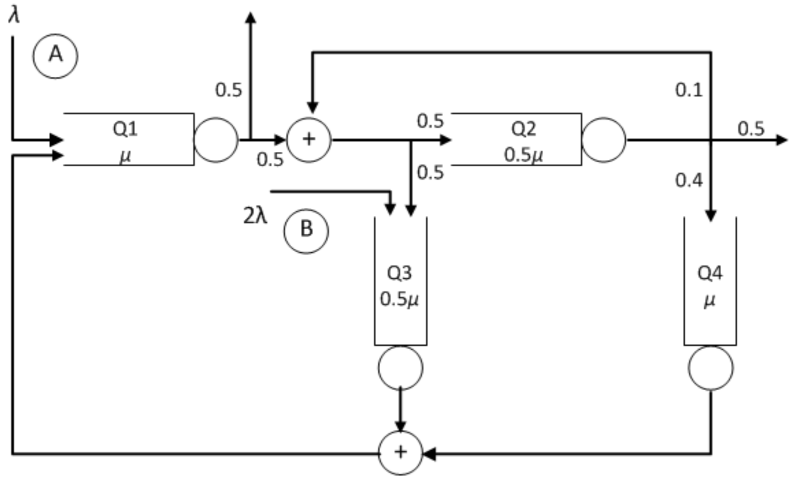
3. Consider a FCFS $M^{[X]}/G/1$ queue where the arrivals come in batches where there are either one or two jobs in the batch with equally likely probabilities, i.e. P{Batch Size = 1} = P{ Batch Size = 2} = 0.5. The **first** job of the batch has a random service time with its n^{th} moment given to be α(*n*) and L.T. of its pdf as $L_\alpha(s)$. The **second** job of the batch (if any) has a random service time with its n^{th} moment given to be β(*n*) and L.T. of its pdf as $L_\beta(s)$. *The two random variables are independent of each other.*

(a) What will be the mean queueing delay for an arbitrary job (first or second in a batch) and the L.T. $L_{Wq}(s)$ of its pdf? **[3+4]**

(b) What will be the mean queueing delay observed by the second job in a batch with two jobs. **[3]**

Hint: Standard results for the basic M/G/1 queue (using standard notation), may be stated properly and used directly, i.e. there is no need to derive these if you can state them correctly.

4. Consider the open network of single server queues with service rates as shown. Jobs enter the system either from **A** with rate λ or from B with rate 2λ.

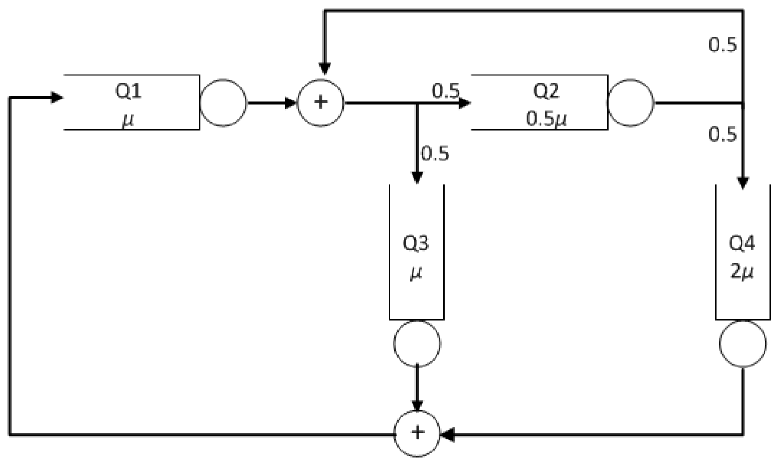


(a) What is the condition for the system to be stable? **[3]**

(b) What is the mean transit time through the system for any job arrival when **λ=0.1 and μ=1**? **[3]**

(c) For **λ=0.1 and μ=1**, what will be the transit time through the system for a job entering the network at B? **[4]**

5. Consider the following closed network of single server queues where the service rates are as shown with M=4 customers circulating in the network. *To apply Norton's Theorem in this network, consider Q4 to be the target queue*. **Assume μ=1**



(a) Find the **Flow Equivalent Server (FES)** for M=4 for the rest of the network (i.e. Q1, Q2 and Q3). **[5]**

(b) Draw the equivalent network consisting of the FES and Q4 and analyse it to obtain probabilities of finding k jobs in Q4, k=0, 1, 2, 3, 4. **[5]**