EC633 Queueing Systems (2009-10-I)
Mid Term Examination

Total Marks = 120 **Maximum Marks = 100**

1. Do the transient analysis of an M/M/1/1 queue assuming that the queue starts empty at time t=0. Follow the standard notation used in class. **[25]**

2. Consider a M/M/2 queue with two servers A and B with respective service rates μ_A and μ_B where the service times are exponentially distributed. **If an arrival finds the system empty then it gets served by the server which was last busy**. In all other situations, the system behaves like a normal M/M/2 queue, i.e. customers are served in a FCFS fashion by whichever server is available. Arrivals come to the system at rate λ.

(a) Draw a state transition diagram for the system with an appropriate definition of the system states. **[10]**

(b) Solve this for the system state probabilities and use these to get the probabilities of finding k users in the system for k=0,1,2,..... **[10]**

(c) You decide to penalize the servers in proportion to the time that they are idle. What is the ratio in which servers A and B should be penalized? **[5]**

3. Consider a M/M/1 queue where the server goes for **a single vacation** (exponentially distributed with mean β^{-1}) whenever the system goes empty. On return from this vacation, the server starts serving whoever may have arrived during the vacation or stays available for the next arrival. Assume that the arrivals to the queue come at rate λ and the server serves customers at rate μ.

(a) Draw the state transition diagram of the system with an appropriate definition of the system state. **[10]**

(b) Write the balance equations. **[10]**

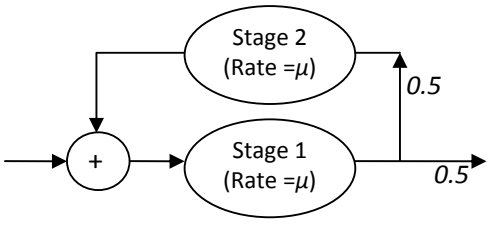
(c) Using the balance equations, compute the probability of the server being on vacation. Give your expression in terms of λ, β and μ. **[15]**

4. Arrivals come in batches of *either one or two jobs* (equally likely) to a single server queue where only a *maximum of two customers* can be in the system at any given instant of time. The batches arrive from a Poisson process with (batch) arrival rate λ and the *WBAS service* strategy is being followed. The server provides service in stages as shown below where each stage has exponentially distributed service times.

(a) What is the overall service time distribution (or its transform)? **[5]**

(b) Draw the state transition diagram with an appropriate definition of the system states. **[10]**

(c) Use this to obtain the state distribution of the system. **[15]**



(d) What is the probability that an arriving batch will not be allowed to enter the system? **[5]**